RADIO CORPORATION OF AMERICA
RCA LABORATORIES

AN ANALYSIS OF FACTORS LIMITING SEISMIC DETECTOR SENSITIVITY

FINAL REPORT

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AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
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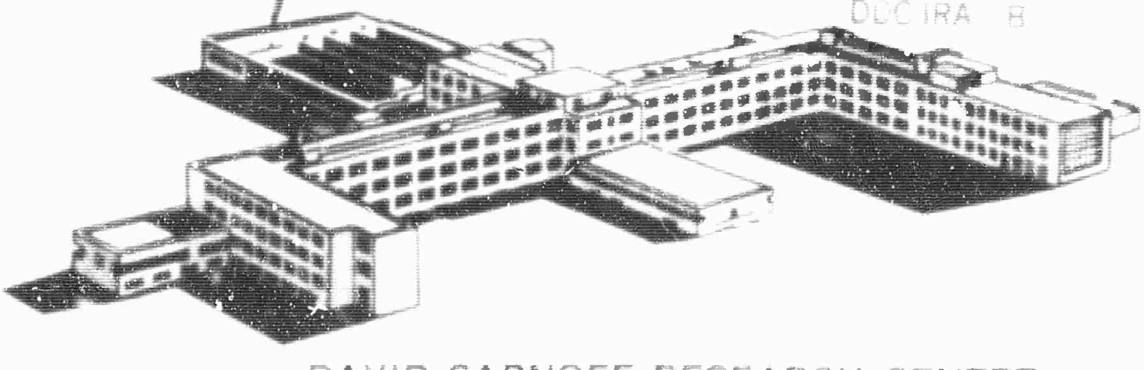

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FINAL REPORT

AN ANALYSIS OF FACTORS LIMITING SEISMIC DETECTOR SENSITIVITY

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I. INTRODUCTION

The study of seismic detector sensitivity has been in progress at RCA Laboratories under contract AF49(638)-1080 since 15 June 1961. The original fifteen month contract period has subsequently been extended for two additional twelve month periods.

The goal which was originally stated for the project was "to determine those factors which limit the sensitivity of seismic detectors and to estimate the maximum obtainable sensitivity with techniques available in the foreseeable future as a function of the required frequency range, limitations imposed upon the size and weight of the mass, the environmental conditions under which the instrument must work and any other parameters which are found to be significant, corroborating these findings wherever possible by experiment."

Five semiannual reports have been issued in the course of this work plus a Technical Summary Report issued at the end of the first fifteen months. The more important results from these reports are summarized in Section II of this final report.

In Section III, noise spectrum measurements are presented for several amplifiers, including the United Electrodynamics GA-220 galvanometer amplifier, the Electrotech SPA-1-1 solid state amplifier and a solid state amplifier obtained from the Weston Observatory. The noise spectrum curves from several amplifiers tested earlier are presented in a form somewhat different from the earlier presentations in order to permit easier comparisons among them.

In Section IV the practical problem of choosing the transducer parameters of the seismometer in order to optimize the signal to noise ratio when driving a given amplifier is discussed and general guidelines are presented.

In Section V signal to noise figures including both thermal noise and amplifier noise, are presented for several seismometer-amplifier combinations including the Johnson-Matheson Vertical seismometer and the Hall-Sears HS-10-1 seismometer and the Texas Instruments RA-1 amplifier, the Electrotech SPA-1-1, the Geotech GPTA and the UED GA-220 amplifier.

In Section VI further discussion of the "detectability" criterion for signals in the presence of noise of various bandwidths is presented. Examples of the detectability of sinusoidal bursts of varying lengths are shown and the effective bandwidth of the eye as a filter of visual records is estimated.

II. SUMMARY OF WORK TO DATE

It is well known among those who are engaged in research of any type that knowledge is seldom gained in the logical and orderly manner in which it is presented in textbooks. It is often more interesting and instructive to follow the actual course of a learning process than an "ad hoc" idealization of the most straightforward procedure. Therefore, this summary of the past work of this contract is presented in a more or less chronological order with some indication of the motivation for various phases of the work given wherever possible.

A. THE BASIC ANALYTICAL APPROACH

The fundamental and unavoidable limitations to sensitivity of seismic detectors appear to be the thermal noise of the mechanical and electrical elements of the seismometer and transducer and the noise of the amplifier, inherent in the active internal processes which provide the amplification. From the outset the basic analytical approach has been to evaluate sensitivity in terms of the ratio of electrical signal voltages and noise voltage spectra appearing at the input terminals of the amplifier. This procedure required that the mechanical elements of the seismometer, i.e., the inertial mass, the spring, and the mechanical losses, be represented by the analogous electrical components and circuits which appear to be present when viewed from the electrical terminals of the transducer. The thermal noise voltage spectrum appearing at the amplifier input terminals could then be calculated conveniently by means of the familiar Nyquist noise relations.

The validity of calculating the thermal noise of mechanical configurations by analysis of the analogous electrical circuit has been established by C. J. Byrne[1] from general thermodynamic considerations, and in Semiannual Technical Summary No. 3 of this contract by deriving the spectral density functions for thermal noise forces and velocities in mechanical configurations from the motion of the molecules of the damping medium.

In the very early work under this contract, following the lead of Wolf,[2] most of the attention was focussed on the thermal noise limitation. Using the technique outlined above, Wolf's analysis was generalized to permit consideration of damping other than critical and the improvement of signal to noise ratios by the use of suitable band limiting filters. Several other methods of improving signal to thermal noise ratio and thereby reducing the required minimum inertial mass were considered.

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[1] C. J. Byrne, "Instrument Noise in Seismometers" Bull. Seis. Soc. Am., Vol. 51, No. 1, January 1961.

[2] Alfred Wolf, "The Limiting Sensitivity of Seismic Detectors, Geophysics, Vol. VII, No. 2, April 1942.

B. REFRIGERATION OF AN EXTERNAL DAMPING RESISTANCE

One of the means considered for reducing the thermal noise was that of refrigerating an external electrical damping resistance. An analysis of the effect of such refrigeration on the thermal noise spectrum of a seismometer was presented in Semiannual Technical Summary No. 2 and subsequently expanded in the Technical Summary Report issued at the end of the first 15 month contract period.

This analysis, which considered only the moving coil type of velocity transducer, showed that temperatures lower than that of liquid nitrogen would be required to achieve a detectable reduction in thermal noise for transducers of typical efficiency.[3]

All of the analysis of thermal noise performed in the course of this contract has assumed a moving coil or moving magnet transducer. This type of transducer was chosen because it lends itself conveniently to analysis, because of its simplicity of construction and large dynamic range, and the fact that it has been used successfully in a great many high sensitivity instruments. Although discussion of the basic theory of operation of several types of capacitor transducers was presented in semiannual report No. 2 along with some rudimentary attempts to estimate sensitivity limits, nothing further has been done with displacement transducers.

C. INITIAL WORK ON FEEDBACK

Some attention was also given in the original proposal and early reports to the possibility of improving the signal to thermal noise ratio of the system by means of feedback around a two-terminal external damping resistor to reduce its effective noise temperature. The feasibility of this idea was found to hinge upon the availability of an amplifier with an inherent internal noise (or "excess noise") smaller than the thermal noise of its input resistance over the desired frequency range.

It was the need to find an amplifier for this feedback scheme that furnished the initial motivation for developing equipment and techniques for measuring the noise spectra of amplifiers. Subsequently, as the work progressed, measurements of amplifier noise spectra made with this equipment made it possible to establish the nature of the sensitivity limitation imposed by the amplifier itself for several amplifiers frequently used with high sensitivity seismometers.

It should be noted that the initial investigations of feedback have indicated that the degree of improvement which is possible with existing amplifiers and seismometers would be small and probably would not justify the additional complication involved.

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[3]It was in the course of this analysis that some of the relationships between transducer efficiency and the volumes of the magnet and coil and the other parameters of the system were recognized. The problem of optimizing the efficiency of transducers of the moving coil or moving magnet type was considered in much greater detail in Semiannual Technical Summary, No. 4. The subject of transducer design and optimization is quite involved, however, and the discussion given in this report treated the problem only in sufficient depth to develop the relations necessary for the noise analysis.

D. NOISE MEASURING EQUIPMENT

The amplifier noise spectrum measuring equipment has been described in considerable detail in the semiannual reports and a brief desc iption should suffice at this point.

The equipment permits the measurement of the mean square noise voltage in a one-third octave band centered at any frequency from about .01 cps to 10 cps. It was recognized at the outset that the accurate measurement of the mean square noise in such narrow bandwidths would require that the mean square voltage be averaged over periods of 6 hours or more. In practice, noise recordings of approximately 15 hours have been made by recording overnight. Therefore, it was felt that the only way of completing a spectrum measurement in a reasonable time period was to record the noise on magnetic tape at a slow speed of approximately 1/6 inch per sec. and play it back at 120 inches per sec. to compress the time scale by a factor of 750 to 1. This time compression converted the frequency range of the noise into the audio band where conventional audio spectrum measuring equipment could be used. To enhance the signal to noise ratio of the measuring equipment the noise to be measured was recorded as a frequency modulation of a 100 cps square wave. On playback at high speed this carrier was demodulated by a discriminator and a true mean square value was determined by use of a vacuum thermocouple. In the course of the project a succession of improvements in this equipment have been made and its reliability is felt to be excellent when proper calibration procedures are followed. Noise spectrum measurements have been repeated to accuracies of 1/2 db over a period of one month.

E. AMPLIFIER NOISE SPECTRA AND EQUIVALENT CIRCUIT REPRESENTATIONS

Noise spectra have been measured for a number of different amplifiers including models typical of most amplifiers used in high sensitivity seismic detector systems. The amplifiers tested are (in chronological order):

1. The Geotech Model 4300 Galvanometer Phototube Amplifier
2. The Honeywell Deviation Amplifier, Model 62-R-360-A-B
3. The Texas Instruments Model RA-0 Amplifier
4. The Texas Instruments Model RA-2 Parametric Amplifier (two units were tested)
5. The United Electrodynamics Model GA-220 Galvanometer Amplifier
6. The Weston Observatory Amplifier
7. The Electrotech Model SPA-1 Amplifier

An effort has been made in a later section of this report to summarize the results of the measurements on the more useful of these amplifiers in a manner somewhat different from that used in the semiannual reports in order to permit easier comparisons among them. In addition, signal to noise ratios of several seismometer-amplifier combinations are presented.

In order to investigate the noise of a seismometer-amplifier combination considering the sensitivity limitation imposed by the total instrument noise, including both thermal noise and amplifier excess noise, it was desirable to represent the excess noise of the amplifier in terms of a single equivalent noise voltage generator at the input of the amplifier. However, in measuring the noise spectrum of the Honeywell Deviation Amplifier it appeared that the magnitude of this equivalent noise voltage source was a function of the source impedance of the amplifier. The explanation for this appeared to be that a component of noise generated by some internal mechanism of the amplifier was appearing at the input terminals and producing a noise current flow in the input circuit which was a function of source impedance.

To obtain a satisfactory and more general equivalent circuit representation for amplifier noise sources which would include this kind of behavior, it was necessary to postulate two equivalent input noise generators, a noise voltage generator in series with the input as before and a shunt current generator. Methods were developed for measuring the spectra of these two noise generators and the correlation which, in general, would exist between them. A discussion of this work was presented in Semiannual Technical Summary No. 3 and in a paper presented by Mr. John Fischer at the conference on Seismic Amplifier Noise which was held at RCA Laboratories in September 1963. It should be pointed out, however, that the noise behavior of the majority of amplifiers used for high sensitivity applications can be adequately represented by a single equivalent noise voltage generator in the input circuit.

F. COMPARISON OF SEISMOMETER NOISE ANALYSES

In Semiannual Technical Summary No. 3 a comparison was made of the seismometer noise analyses of Alfred Wolf, C. J. Byrne, and those done under this contract. The conditions under which the various analyses are valid were catalogued and several important typographical errors in the equations of Wolf's paper were noted. The intent of this discussion was to allay some confusion which has existed over apparent differences among these three seismometer noise analyses. It was pointed out that when applied to equivalent cases all three analyses produce the same results.

G. MINIMUM INERTIAL MASS REQUIREMENT

One of the original tasks of this contract was to determine the minimum value of seismometer inertial mass which would permit the detection of ground motion of given amplitude and frequency. Although the seismometer is physically a simple device, this task is complicated by the many interrelated parameters of a seismometer-amplifier system. The earlier reports issued under this contract contained efforts to identify these parameter relationships and to establish a general mathematical description of the inherent instrument noise of a seismometer system as a function of its parameters. These reports display a continuous shift in emphasis toward recognition of the amplifier noise as a more important factor in determining the sensitivity of the entire system than it had originally been assumed to be.

In Semiannual Technical Summary No. 4, the problem of determining the minimum inertial mass requirement for the seismometer was approached from the standpoint of assuming that the amplifier has been chosen first and then asking the question, "What is the required inertial mass of the seismometer which will detect a given ground velocity over the desired passband when driving this amplifier?"

Mathematical equations were presented and a procedure outlined which permits the minimum inertial mass to be calculated if the equivalent input noise spectrum and input impedance of the amplifier are known. An example of the use of this procedure was presented, calculating the limiting sensitivity of a system comprised of a Jersey Production Borehole Seismometer and a Texas Instruments RA-2 Parametric Amplifier.

In order to ease the complexities of the mathematics and to simplify the presentation of the results, a set of restrictions were placed on the characteristics of the seismometer-amplifier combinations to which this analysis is applicable. These restrictions were listed in detail in the report and for the most part they were not confining. The most troublesome one, however, was that the reactive components of the input impedance of the amplifier were assumed to be negligible within the passband of the system. In considering seismometer-galvanometer amplifier combinations this restricts the use of the results to the lower frequency end of the passband. However, because of the 1/f noise, which seems always to be present, this is likely to be the point in the passband at which the sensitivity is most severely limited anyway.

H. THE "DETECTABILITY" CRITERION

In the analysis of instrument noise as a limitation to the detection of signals from seismic disturbances it is customary to compare the rms value of the output signal in response to a sinusoidal ground velocity with the rms value of the noise determined for the entire passband of the seismic detection system. Wolf (1942) has stated, based on his experience with galvanometers, that the rms value of the signal must exceed the rms value of the noise by a factor of approximately $\sqrt{20}$ in order to insure detectability. This criterion has been used in the subsequent analyses of Byne (1961) and those performed under this contract.

However, because the detectability criterion directly affects the estimate of the system sensitivity, an investigation of Wolf's criterion was conducted and preliminary results were reported in Semiannual Report No. 5. In this report chart traces showing sinusoidal signals in the presence of noise of various bandwidths were presented. These traces demonstrated that, regardless of noise bandwidth, Wolf's criterion is extremely conservative and that under most conditions it is probably high by a factor of 2. Additional work on the detectability of sinusoidal bursts in the presence of noise are presented in a later section of this report.

III. AMPLIFIER NOISE MEASUREMENTS

In the previous reports of this contract the noise performance of the various amplifiers measured was presented in terms of an equivalent input noise voltage source in series with the source impedance connected to the amplifier input. This form of noise representation includes the effects of the thermal noises of the amplifier input impedance and the source impedance used during noise measurement. As a consequence, this form of noise representation has disadvantages when used to compare the noise performance of several amplifiers. To facilitate the comparison of the noise characteristics of different amplifiers a change in the form of presentation of the noise performance of the amplifiers measured was made.

The noise performance of the amplifiers measured other than galvanometer type amplifiers will be presented in terms of an equivalent input noise voltage source in series with the input of a noiseless amplifier having an infinite input impedance. The frequency response of this noiseless amplifier is identical to that of the noisy amplifier. The input impedance of the actual amplifier will be represented by a separate impedance exhibiting the appropriate thermal noise. A diagram of this noise representation is shown in Fig. 1.

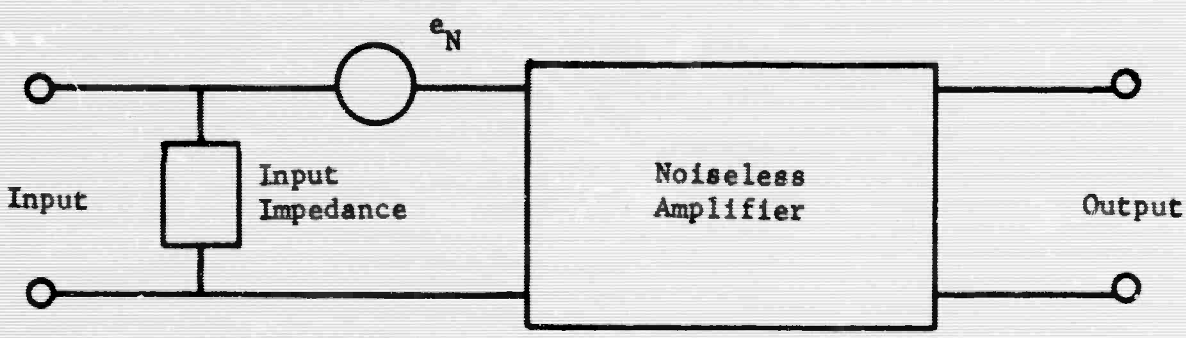

Fig. 1. Amplifier Noise Equivalent Circuit.

Using the form of noise presentation outlined above the total equivalent input noise spectrum may be calculated using Eq. (1) below.

$$\overline{e^2_{Total}} = \overline{e^2_N} + \overline{e^2_{Thermal}} \tag{1}$$

where $\overline{e^2_{Total}}$ is the total equivalent mean square input noise voltage spectrum level, $\overline{e^2_N}$ is the amplifier excess equivalent mean square input noise voltage spectrum level, and $\overline{e^2_{Thermal}}$ is the thermal noise mean square voltage spectrum level for the parallel combination of the source impedance and the amplifier input impedance. The output total noise spectrum may be obtained by multiplying the input total noise spectrum by the amplifier gain.

For galvanometer type amplifiers the noise presentation will again be made in terms of a voltage source e_N in series with the input of a noiseless amplifier having infinite input impedance. In this case, however, the noise source and noiseless amplifier will be preceded by a two-terminal-pair network representing the input circuit of the amplifier and exhibiting the appropriate thermal noise. The frequency response of the noiseless amplifier will be equal to the gain of the noisy amplifier divided by the voltage transfer function of the two-terminal-pair input circuit. A diagram of this noise representation is shown in Fig. 2.

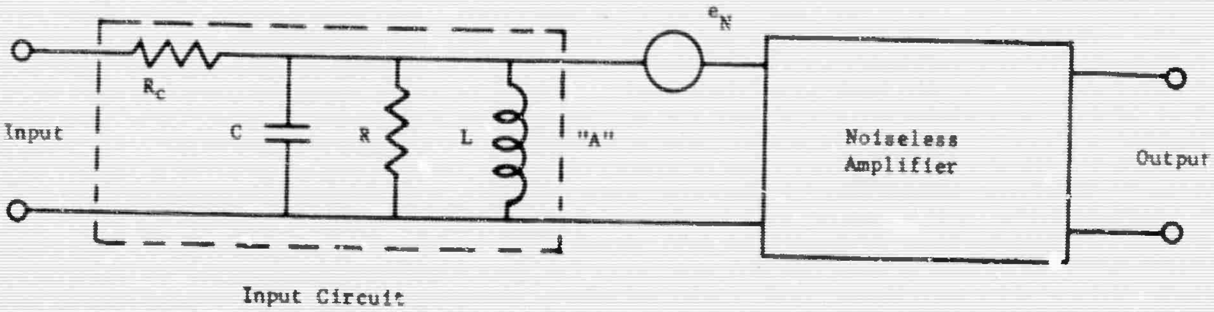

Fig. 2. Galvanometer Type Noise Equivalent Circuit.

Employing this form of noise presentation the total output noise spectrum may be calculated as shown in Eq. (2) below

$$\overline{e^2_{\text{Total Out}}} = |A_N|^2 \left[\overline{e^2_N} + \overline{e^2_{\text{Thermal}}}\right] \tag{2}$$

where $\overline{e^2_{\text{Total Out}}}$ is the total mean square output noise voltage spectrum level, $\overline{e^2_N}$ is the mean square excess noise voltage spectrum level of the amplifier referred to the input of the noiseless amplifier, $\overline{e^2_{\text{Thermal}}}$ is the mean square thermal noise voltage spectrum level of the input circuit and source impedance combination measured at the input of the noiseless amplifier (point A of Fig. 2), and A_N is the gain of the noiseless amplifier.

A disadvantage of presenting the excess noise of a galvanometer type amplifier in terms of the voltage source e_N of Fig. 2 is that "1/f" noise in the amplifier following the galvanometer is not immediately apparent. Because of the integral relationship between galvanometer velocity and position which is included in the gain of the noiseless amplifier of Fig. 2, a flat noise spectrum for the voltage source e_N may indicate "1/f" noise in the portion of the amplifier following the galvanometer.

2. NOISE SPECTRUM MEASUREMENTS OF UNITED ELECTRODYNAMICS, INC. MODEL GA-220 GALVANOMETER-PHOTOSENSITIVE AMPLIFIER

The United Electrodynamics, Inc. Model GA-220 Galvanometer-Photosensitive Amplifier employs an input galvanometer, a photosensitive element, and a solid state amplifier. In this amplifier signal is supplied to the input galvanometer producing a galvanometer rotation. A light beam falling on the galvanometer mirror is deflected by the rotation of the galvanometer mirror, changing the position of a light spot on the photosensitive element. The photosensitive element is made up of a metallic strip and a strip of resistive material placed parallel with a photoconductor between them. A voltage is applied between the ends of the resistive strip, and when a spot of light illuminates a small area of the photoconductor, the photosensitive element operates as a potentiometer in which the wiper contact is made through the illuminated section of the photoconductive material to the metallic strip. The output from the photosensitive element is a function of the position of the light spot and therefore a measure of the galvanometer deflection. The output from the photosensitive element is amplified in a transistor amplifier and supplied to the output terminals. The gain of the Model GA-220 Amplifier is due primarily to the amplifying property of the optical lever utilized in the optical system made up of the light beam source, the galvanometer, and the photosensitive element. A small amount of gain is supplied by the transistor amplifier.

The nominal voltage gain of the Model GA-220 Amplifier with a balanced output is specified to be 4000,000. Measurements on this amplifier were made using single ended unbalanced output and consequently the gain was reduced by a factor of two. Plug-in band pass filters in the frequency ranges of 0.8 cps to 3.0 cps, 0.8 cps to 5.0 cps, and 0.8 cps to 12.5 cps are supplied with the amplifier to allow modification of the amplifier pass band.

Input impedance measurements were made on the Model GA-220 Amplifier. An equivalent input circuit is shown in Fig. 3.

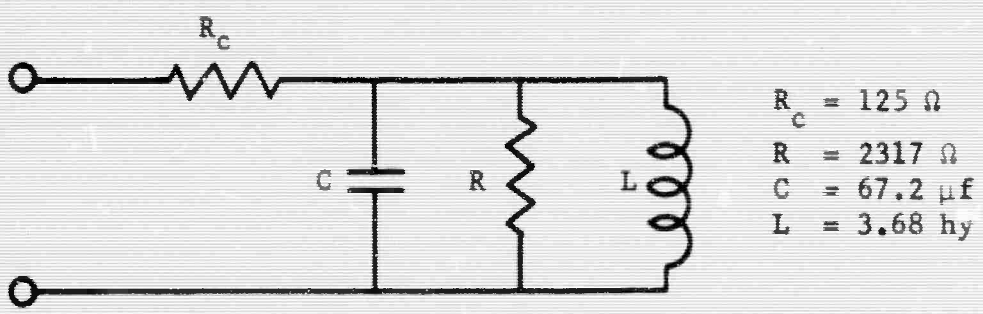

Fig. 3. Input Impedance of United Electrodynamics Model GA-220 Amplifier.

The results of the input impedance measurement are: the galvanometer winding resistance, R_c, equals 125 ohms; the mechanical loss resistance, R, equals 2317 ohms; the equivalent capacitance, C, equals 67.2 μf; the equivalent inductance, L, equals 3.68 hy; and the undamped resonant frequency equals 10.13 cps.

Gain measurement were made on the Model GA-220 Amplifier using the 0.8 cps to 12.5 cps filter and without the bandpass filter. The results of these measurements are shown in Fig. 4. The noise equivalent circuit used to describe the noise performance of the Model GA-220 Amplifier is of the type shown in Fig. 2. The gain of the noiseless amplifier was calculated and the results of these calculations are shown in Fig. 5.

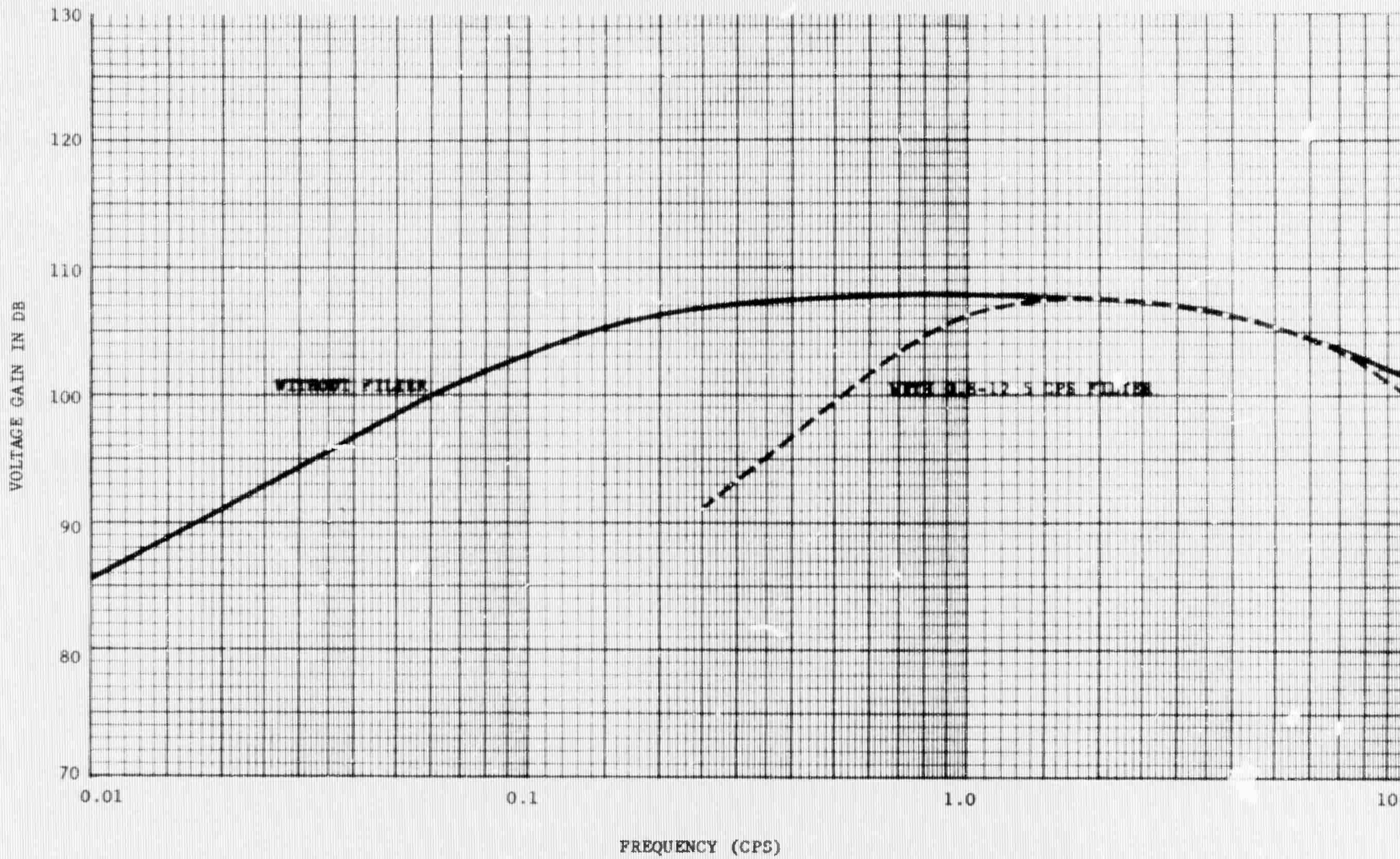

Fig. 4. United Electrodynamics Model GA-220 Amplifier Gain.

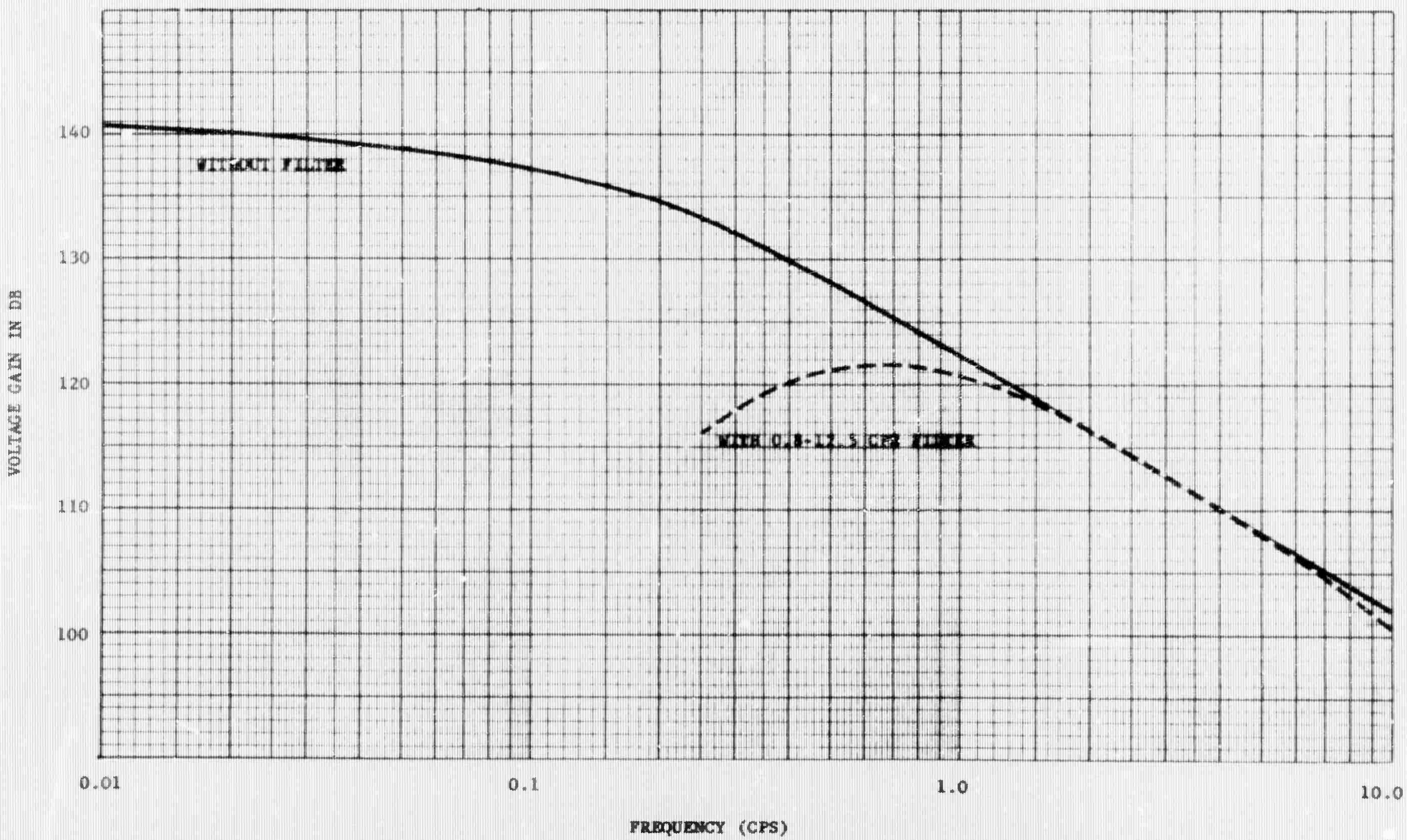

Fig. 5. Noiseless Amplifier Gain for United Electrodynamics Model GA-220 Amplifier.

Noise spectrum measurements were made on the Model GA-220 Amplifier without the band-pass filter and using the filter with a band pass of 0.8 to 12.5 cps. For each of these frequency response conditions noise spectrum measurements were made using an input galvanometer as supplied with the amplifier and a source resistance of 88 ohms. This condition corresponds to a galvanometer damping of 0.6 of critical. Noise measurements for each of the frequency response conditions were also made with a dummy galvanometer replacement for the actual galvanometer. The dummy galvanometer substitutes a fixed mirror in the system in place of the movable galvanometer mirror. The amplifier excess noise voltage spectrum as measured using the galvanometer and the 88 ohm source resistance and that measured using the dummy galvanometer are substantially identical. The measured amplifier excess noise voltage spectrum referred to the input of the noiseless amplifier for the Model GA-220 Amplifier is shown in Fig. 6.

The spectrum curve for the amplifier using the 0.8 cps to 12.5 cps band-pass filter indicates the noise to be higher in the low frequency portion of the curve than that of the amplifier without the filter. Measurements made at frequencies higher than 10 cps indicate a similar increase in noise spectrum level for the amplifier using the filter over that for the amplifier without the filter. These results show that the noise introduced by the active filter is negligible in the frequency passband of the filter. In the frequency stop bands of the filter where the input noise and the noise of the early stages of the amplifier as well as the signal have been attenuated, however, the noise of the filter is significant.

In the course of measuring the characteristics of this amplifier a gain measurement of the amplifier was made with the galvanometer replaced by an interchangeable galvanometer produced by the same manufacturer but having an undamped natural frequency of 1670 cps instead of 10.13 cps. The results of this measurement in the frequency range between 1 cps and 1000 cps is shown in Fig. 7. It is interesting to note that over the major portion of the frequency range under consideration the gain is decreasing with increasing frequency at a rate of 3 db per octave. This result is not accounted for by the response of the galvanometer since the undamped resonant frequency is too high. The rate of decrease of gain indicates a distributed resistance-capacitance type effect such as might be introduced by the finite response time of the photoconductor of the photosensitive element. It should be noted that this decrease in gain extends into the frequency band below 10 cps.

Another interesting characteristic of the Model GA-220 Amplifier was found during the testing of this amplifier. Gain measurements revealed the small signal gain of this amplifier to be a function of the quiescent position of the light spot on the surface of the photosensitive element. A measurement of small signal gain as a function of light spot position was made and the results of this measurement are shown in Fig. 8. The ordinate of this graph is relative amplifier gain and the abscissa is an arbitrary scale proportional to the position of the light beam along the active surface of the photosensitive element. This measurement was made by measuring the output signal for an input made up of a low level signal plus a dc bias. The dc

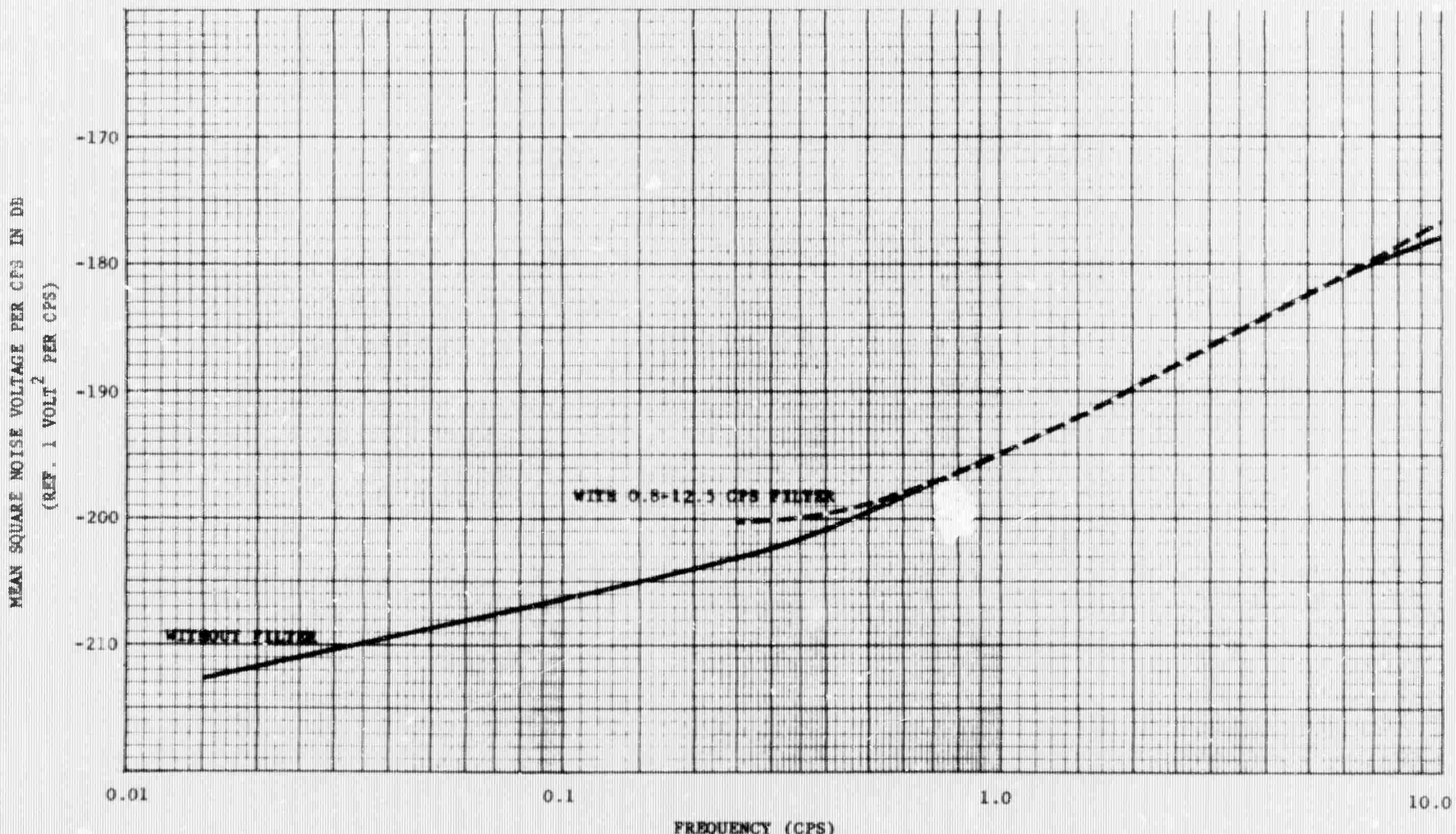

Fig. 6. Equivalent Excess Noise Voltage Spectrum of United Electrodynamics Model GA-220 Amplifier.

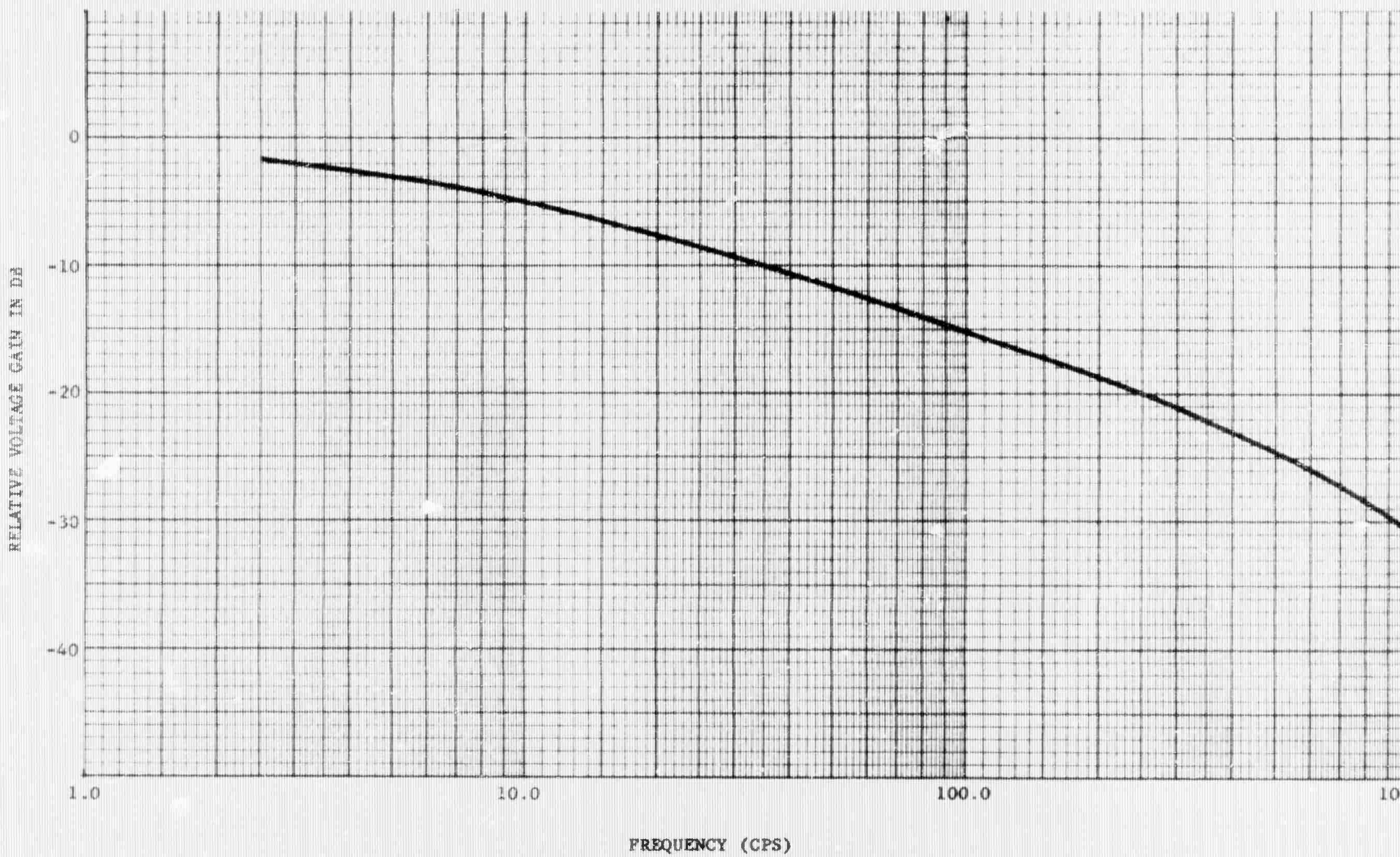

Fig. 7. Relative Gain of United Electrodynamics Model GA-220 Amplifier with High Frequency Galvanometer.

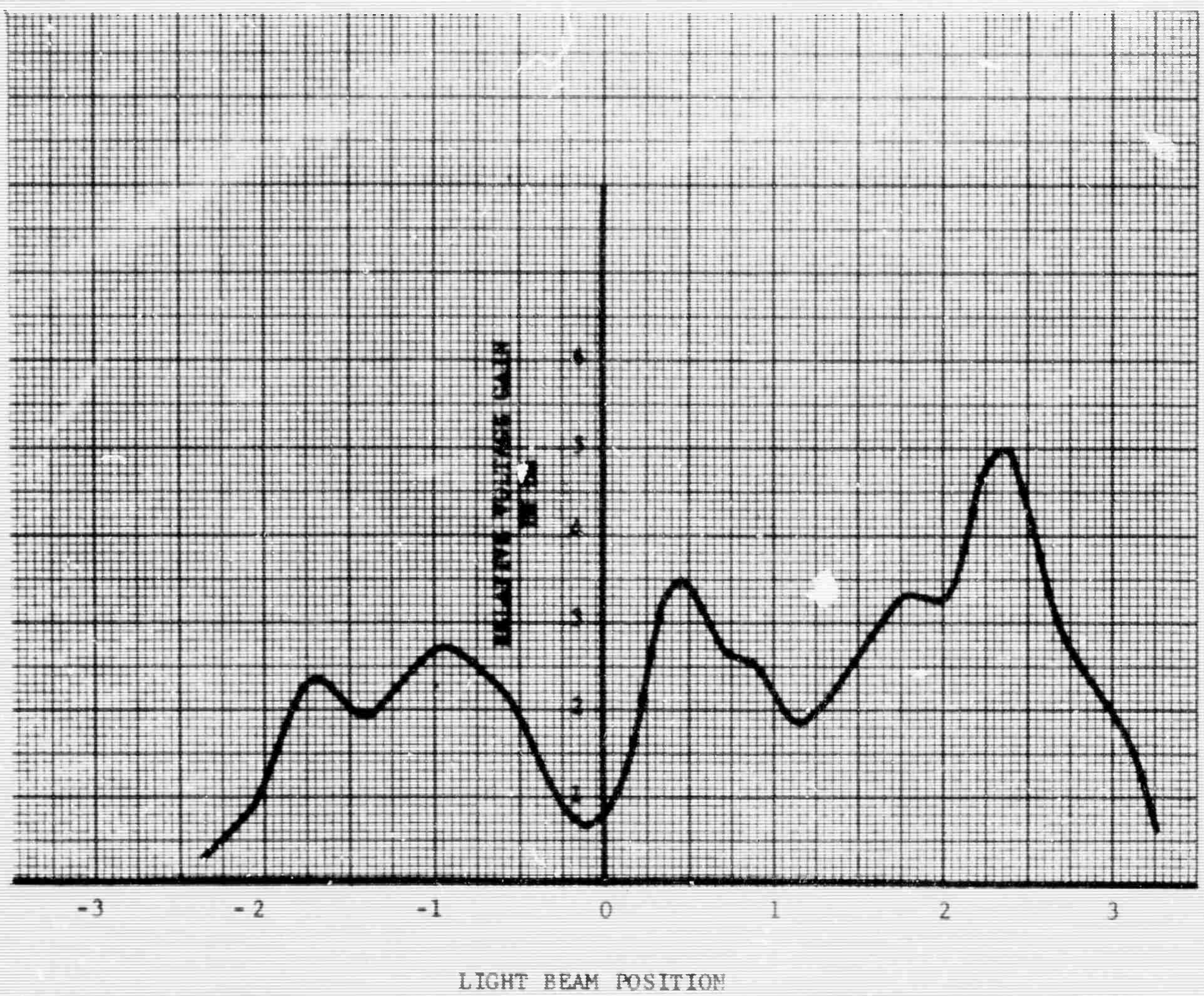

Fig. 8. Relative Gain of United Electrodynamics Model GA-220 Amplifier as a Function of Light Spot Position.

bias was changed to obtain a change in the quiescent position of the light beam. The results of this measurement indicate irregularity in the characteristics of the photosensitive element which may occur in the resistive strip, in the photoconductor, in the metallic strip, or in a combination of these component parts.

The noise performance of the Model GA-220 Amplifier in the frequency range of 0.01 cps to 10.0 cps may be represented by the equivalent circuit of Fig. 9. The results of the measure-

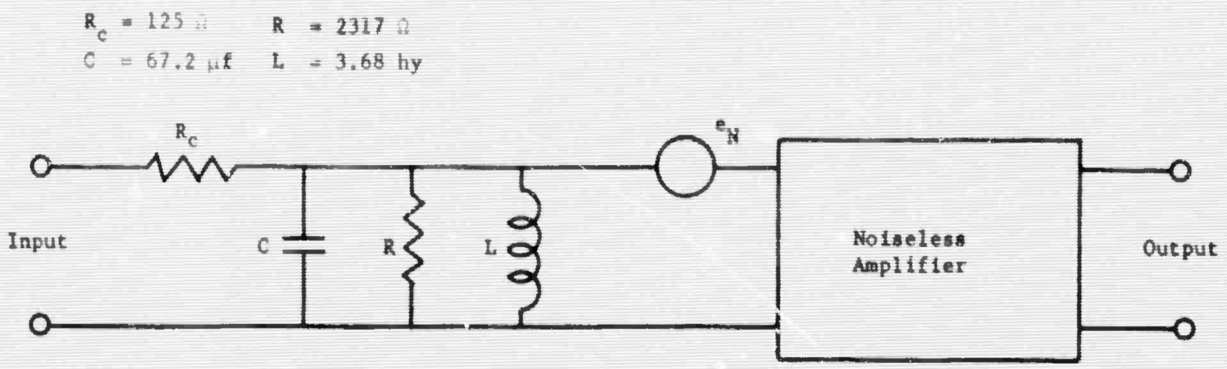

Fig. 9. Noise Equivalent Circuit of United Electrodynamics Model GA-220 Amplifier.

ments made on this amplifier show the spectrum level of the equivalent excess noise voltage generator e_N to be that presented in Fig. 6 and the gain of the noiseless amplifier to be that given in Fig. 5.

B. NOISE SPECTRUM MEASUREMENTS OF WESTON OBSERVATORY AMPLIFIER

Measurements were made of the noise performance of a solid state amplifier used in seismic instrumentation at Weston Observatory.

Measurements of amplifier input impedance show this impedance to be capacitive. The resistive component of input impedance shunting the input capacitance was quite high and its effect is insignificant compared to the capacitive component. The input capacity was measured to be 0.216 μf. The equivalent circuit of this input impedance is shown in Fig. 10.

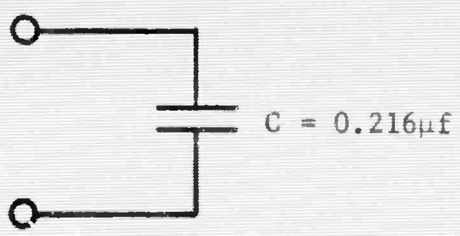

Fig. 10. Input Impedance of Weston Observatory Amplifier.

The measured gain of this amplifier as a function of frequency is shown in Fig. 11.

Fig. 12. Equivalent Input Excess Noise Voltage Spectrum of Weston Observatory Amplifier.

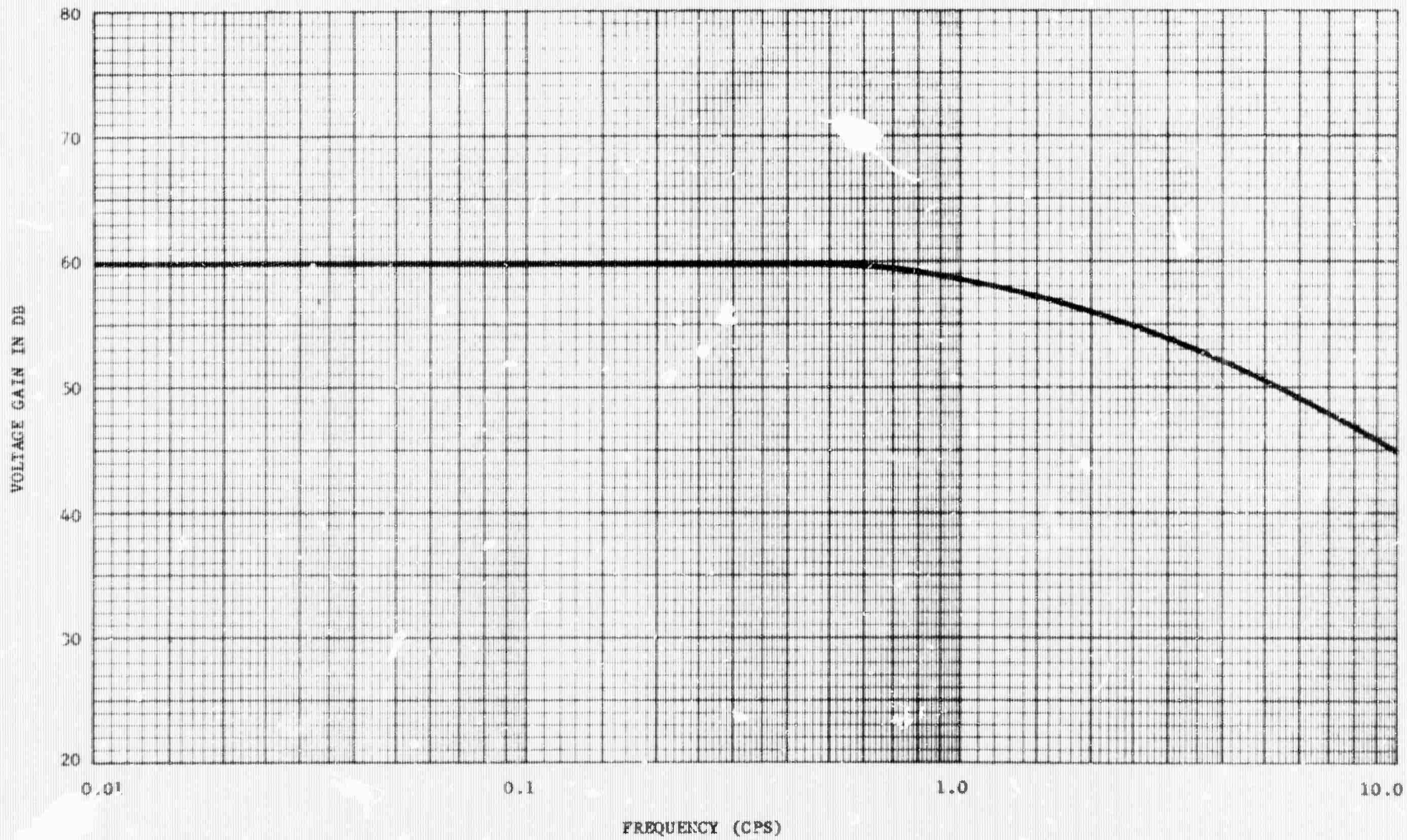

Fig. 11. Weston Observatory Amplifier Gain.

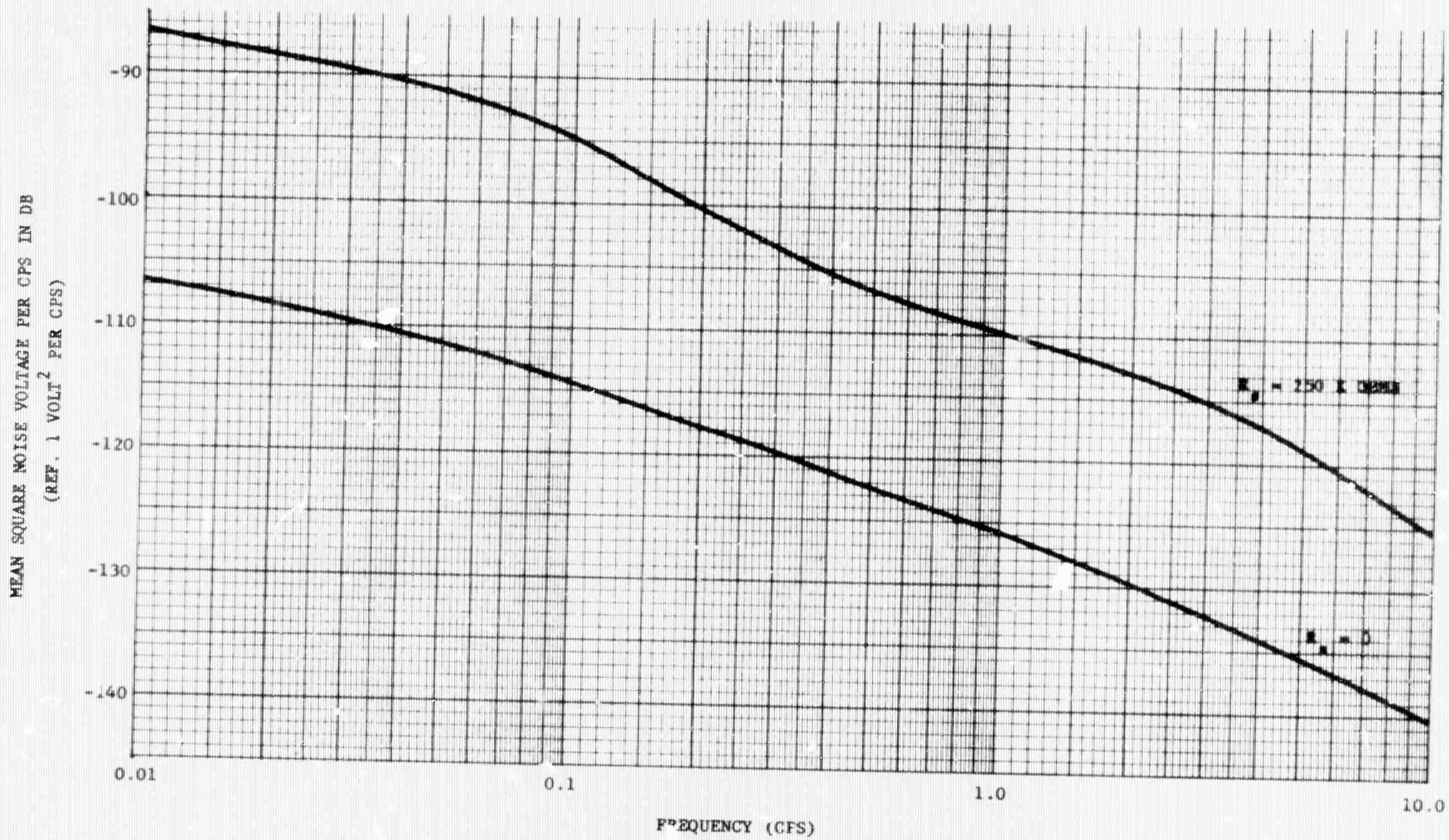

Fig. 12. Equivalent Input Excess Noise Voltage Spectrum of Weston Observatory Amplifier.

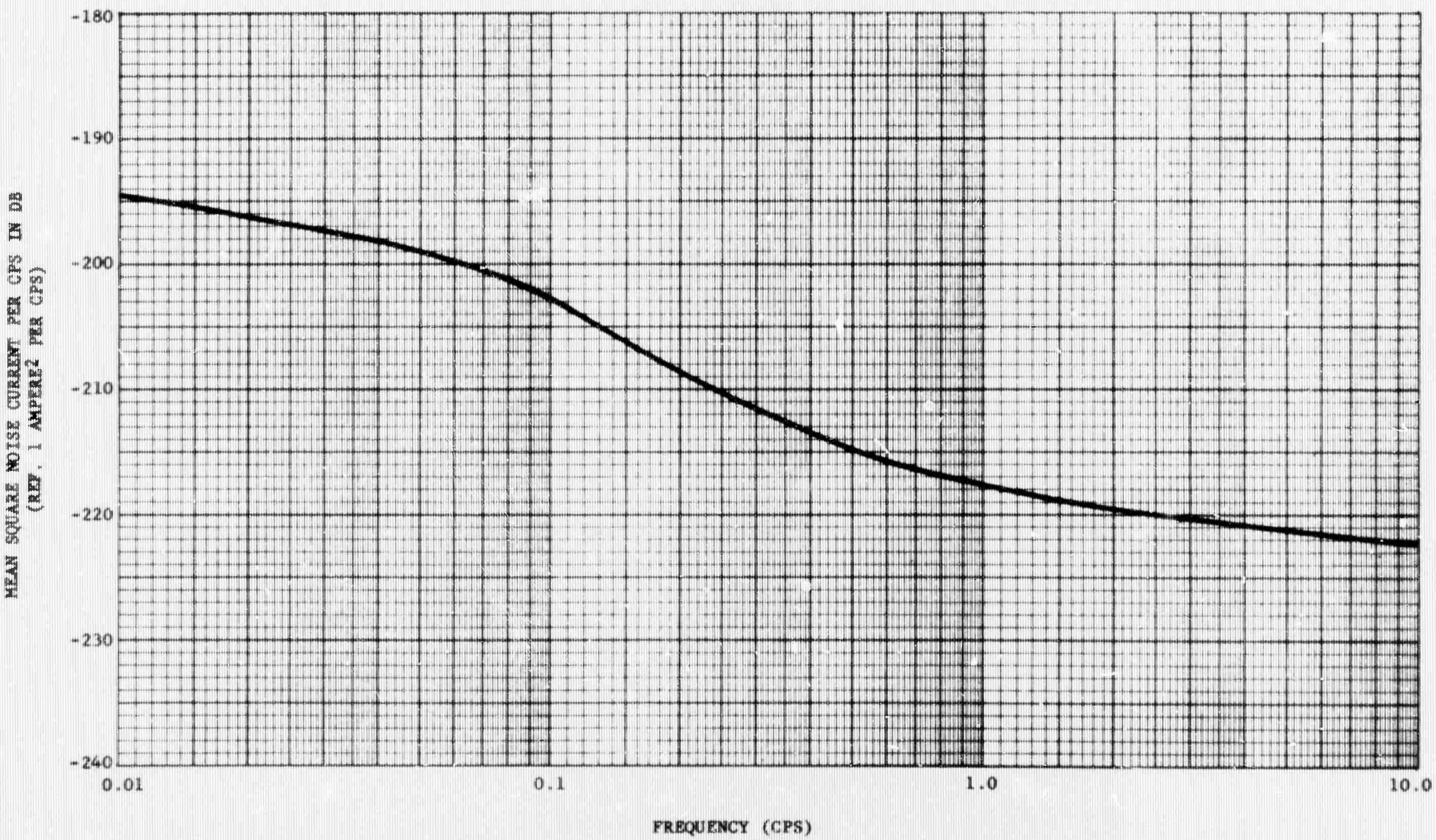

Fig. 13. Equivalent Input Excess Noise Current Spectrum of Weston Observatory Amplifier.

Equivalent input noise voltage spectrum measurements were made on this amplifier for source resistances of zero ohms and 250,000 ohms. The results of these measurements are shown in Fig. 12. These curves exhibit "1/f" type noise characteristic over the entire frequency range of measurement. The noise spectrum level is also quite high with the lowest point on the curve corresponding to the thermal noise spectrum level of a 630,000 ohm resistor. The capacitive input reactance at the same frequency is 73,800 ohms.

The equivalent input noise voltage spectrum level measured for this amplifier using a source resistance of 250,000 ohms is considerably higher than either the thermal noise of the source resistance or the zero source impedance equivalent input noise voltage spectrum of the amplifier. This result indicates the presence of an equivalent input noise current source. Using the equivalent input noise voltage spectrum for a source resistance of 250,000 ohms, the measured amplifier input impedance, and the known 250,000 ohm source resistance an equivalent input noise current spectrum may be calculated. The results of such a calculation is shown in Fig. 13.

The measured noise information on this amplifier is not sufficient to determine the possibility of correlation between the equivalent input noise voltage source and the equivalent input noise current source. In view of the fact that the measured noise spectrum level of this amplifier is quite high no further noise tests were performed on this amplifier. Assuming that the noise voltage and current sources are uncorrelated, any calculation of total input amplifier noise spectrum level will be low by a maximum of only 3 db, if positive correlation exists. A negative correlation of course would present the possibility of partial amplifier noise cancellation for a judicious choice of source impedance.

Using the results of the measurements performed on this amplifier and assuming no correlation between the input noise voltage and current sources, the noise performance of the Weston Observatory Amplifier may be represented by the equivalent circuit of Fig. 14. The gain

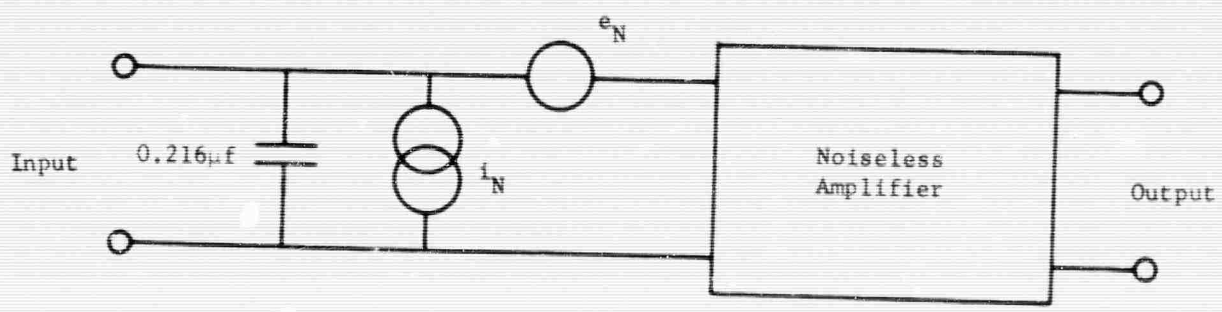

Fig. 14. Noise Equivalent Circuit of Weston Observatory Amplifier.

of the noiseless amplifier is that shown in Fig. 11. The spectrum of the equivalent input noise current source i_N is presented in Fig. 13 and that of the equivalent input noise voltage source e_N is the spectrum for zero source impedance presented in Fig. 12.

C. NOISE SPECTRUM MEASUREMENTS OF THE ELECTROTECH SHORT PERIOD AMPLIFIER SYSTEM MODEL SPA-1-1

Measurements were made of the noise performance in the frequency range of 0.01 cps to 10 cps of the Electrotech Model SPA-1-1 Short Period Amplifier System.

The Model SPA-1-1 Amplifier is a solid state transistor amplifier employing a diode chopper to produce a carrier signal prop-.tional to the low frequency input signal. The carrier signal is amplified, demodulated, and the resulting signal is further amplified before appearing at the output terminals. The nominal voltage gain of the Model SPA-1-1 is 1×10^6 and the nominal input impedance is 2200 ohms. This amplifier has an active low pass filtor with seven cutoff frequencies ranging from 5 cps to 30 cps which may be switched in to modify the high frequency response of the amplifier. Two outputs are available from this amplifier, one of which has an output 30 db below the other.

The results of the noise spectrum measurements made of the Model SPA-1-1 Amplifier are shown in Fig. 15. Noise spectrum measurements were made for a number of different conditions of input and output including, (1) short circuited input without the low pass filter and measuring the noise at the zero db output, (2) short circuited input without filter and measuring the noise at the −30 db output, (3) short circuited input with the 13 cps low pass filter and measuring noise at the −30 db output, (4) 500 ohm input source resistance without filter and noise measured at the −30 db output, (5) 2500 ohm input resistance without filter and noise measured at the −30 db output, and (6) short circuited input without filter with an internal attenuation of 42 db and noise measured at the −30 db output. A measured gain curve for the amplifier in the frequency range of the measurements made is shown in Fig. 16.

The input impedance of the Model SPA-1-1 Amplifier may be determined from the equivalent input circuit for the amplifier shown in Fig. 17. The shunt capacitors effect the input impedance of the amplifier in the frequency range above 60 cps. The frequency at which the shunt capacitive reactance is equal to the amplifier input resistance is variable depending upon

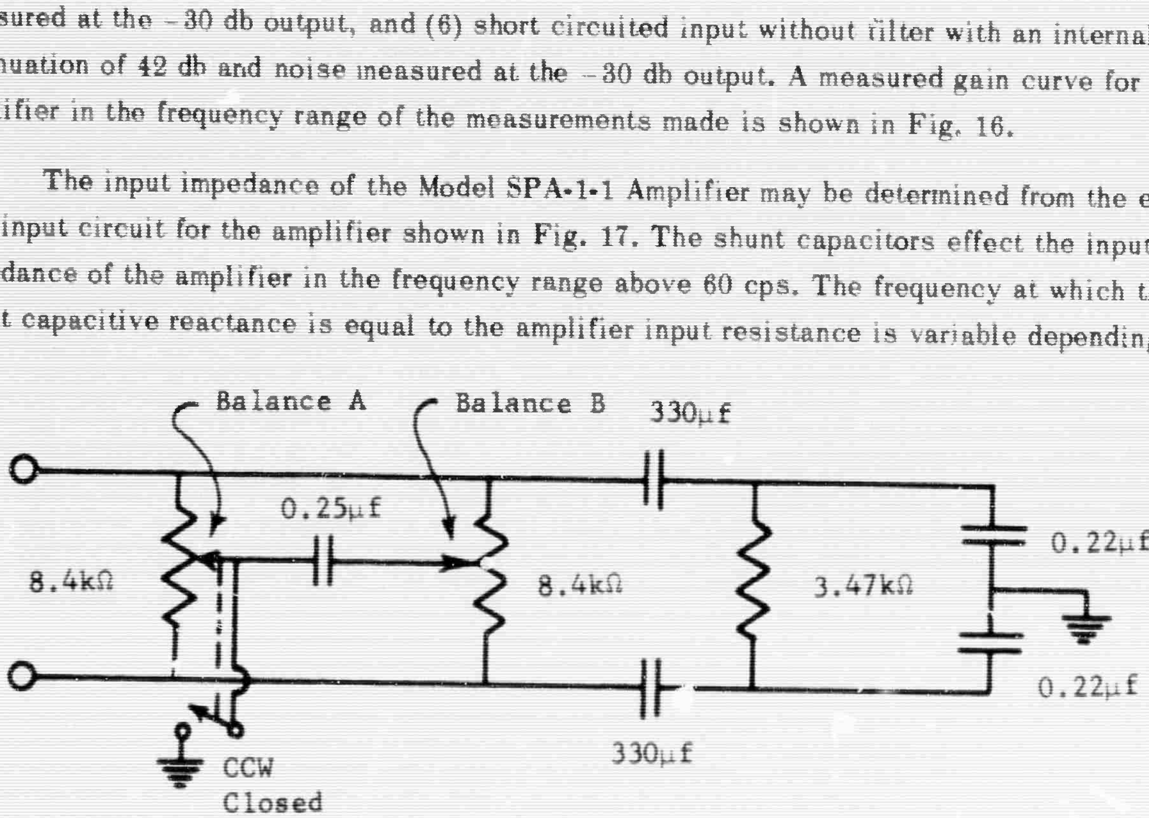

Fig. 17. Input Impedance of Electrotech Model SPA-1-1 Amplifier.

Fig. 15. Equivalent Input Excess Noise Voltage Spectrum of Electrotech Model SPA-1-1 Amplifier.

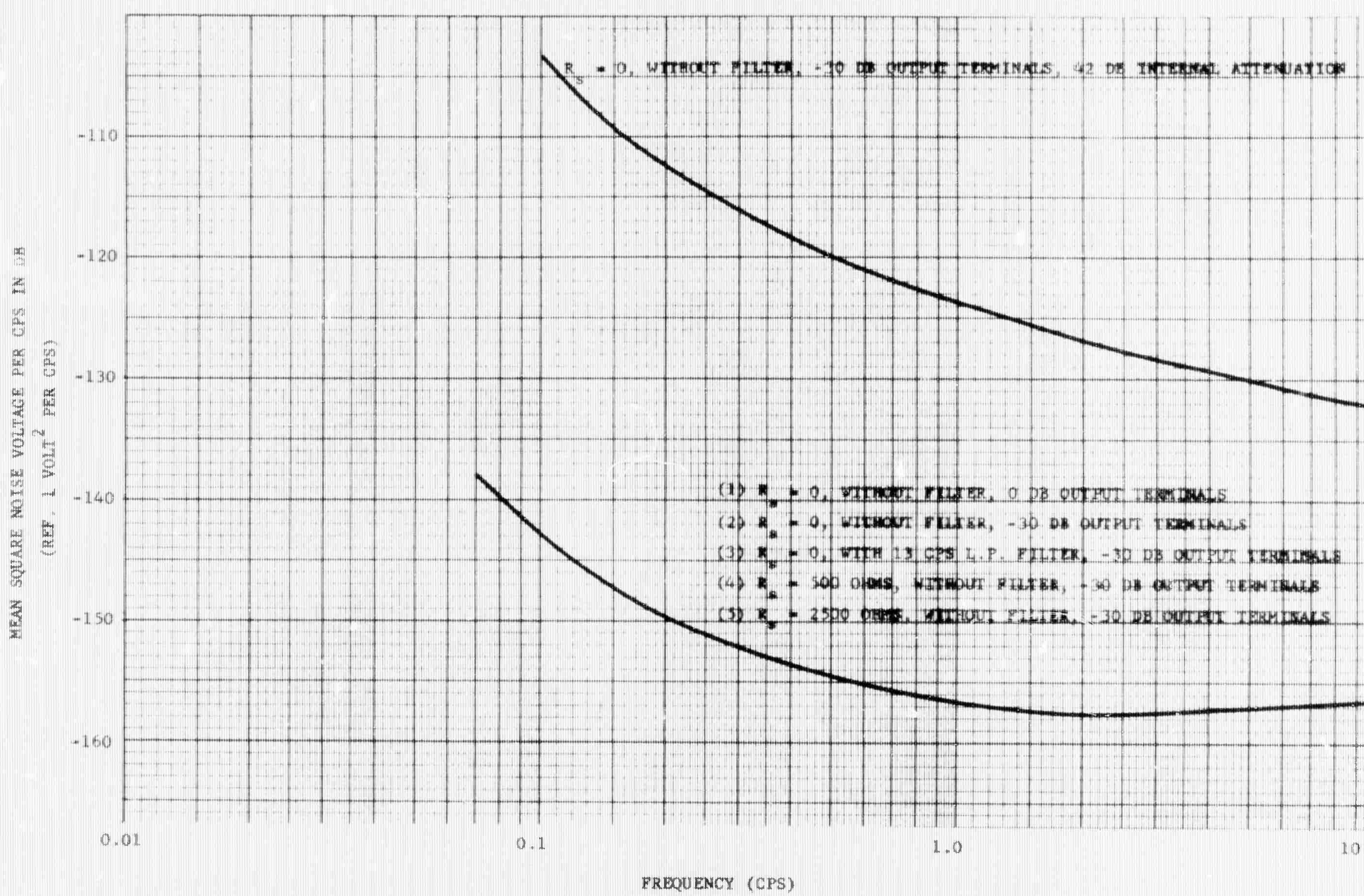

Fig. 15. Equivalent Input Excess Noise Voltage Spectrum of Electrotech Model SPA-1-1 Amplifier.

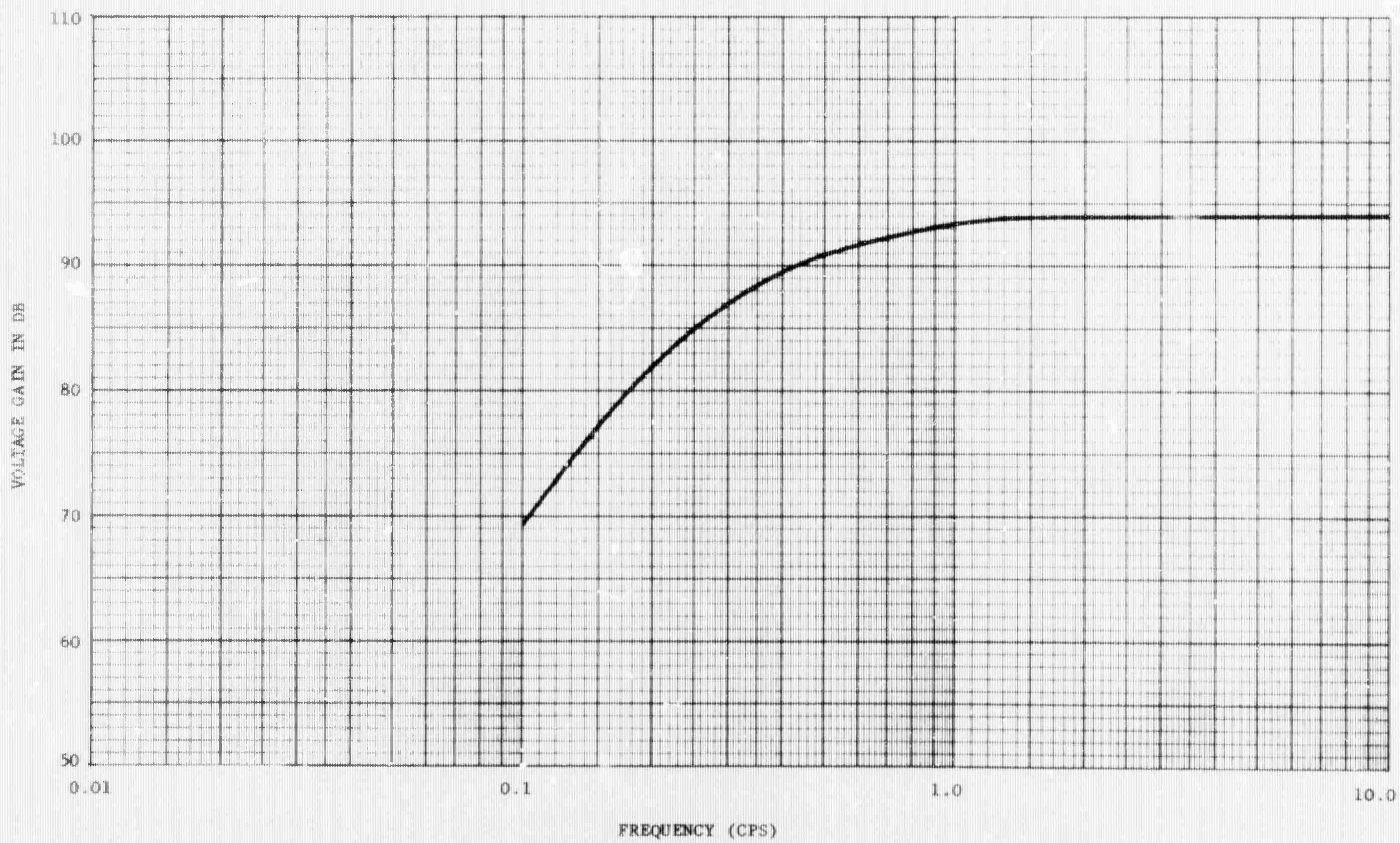

Fig. 16. Electrotech Model SPA-1-1 Amplifier Gain.

the settings of the balance controls. The series blocking capacitors have an effect on the input impedance in the vicinity of 0.5 cps. The measured input impedance at a frequency of 40 cps is 1000 ohms. This impedance increases to a value of 4200 ohms as the frequency is decreased below 0.5 cps.

The noise spectrum curves for this amplifier shown in Fig. 15 exhibit an increasing spectrum level with decreasing frequency. This characteristic of the noise spectrum curves is due to the low frequency cutoff produced by the series blocking capacitors shown above in the equivalent input circuit of the amplifier and to other low frequency cutoffs in the amplifier and possibly also the result of "1/f" noise sources within the amplifier.

Measurements show the noise spectrum of the Model SPA-1-1 Amplifier to be identical for the condition of short circuited input, without the low pass filters, and using the zero db output and −30 db output respectively. This result indicates that those portions of the amplifier circuit unique to the zero db output and to the −30 db output introduce a negligible amount of noise in the Model SPA-1-1 Amplifier. The maximum useable signal obtainable from either output is approximately the same while the noise from the −30 output is about 30 db less than the noise from the zero db output due to the difference in gain. The resulting dynamic range is therefore larger when using the −30 db output than when using the zero db output.

Noise performances of the Model SPA-1-1 Amplifier measured for input source resistances of 500 ohms and 2500 ohms using no filter and the −30 db output are identical to that measured for a short circuited input. This result indicates that no equivalent input noise current source is necessary to represent the noise performance of this amplifier.

The result of a noise measurement made for the condition of short circuited input, without filter, using the −30 db output, and with the internal amplifier attenuator set at 42 db is shown in Fig. 15. The spectrum level of the equivalent input noise voltage source for this condition of measurement is seen to be considerably greater than that for the other measuring conditions. From this result it is concluded that a considerable amount of the noise of this amplifier originates in that portion of the circuit following the attenuator. In view of this result the user of this amplifier is cautioned against using the internal amplifier attenuator of the Model SPA-1-1 Amplifier in an application where minimum amplifier noise is necessary.

Amplifier input noise characteristics for the condition of short circuited input and using the −30 db output and the internal low pass filter with a cutoff frequency of 13 cps are identical to those for the condition without the low pass filter within the frequency range of measurement. Noise measurements were made for frequencies above 10 cps and the results indicate no change in the equivalent input noise voltage for frequencies in the stop band of the filter. From this result it can be concluded that the low pass filter circuit in the amplifier contributes no significant additional noise to the SPA-1-1 Amplifier.

The minimum spectrum level of the measured equivalent input noise voltage source for the SPA-1-1 Amplifier is approximately equal to the spectrum level of the thermal noise of a 10,000 ohm resistor. Since this resistor is larger than the input impedance of the amplifier we see that the amplifier excess noise will always be greater than the thermal noise at the amplifier input in any application.

As a result of the noise measurements on the Model SPA-1-1 Amplifier in the frequency range of 0.01 cps to 10.0 cps the noise performance of this amplifier may be represented by the equivalent circuit of Fig. 18.

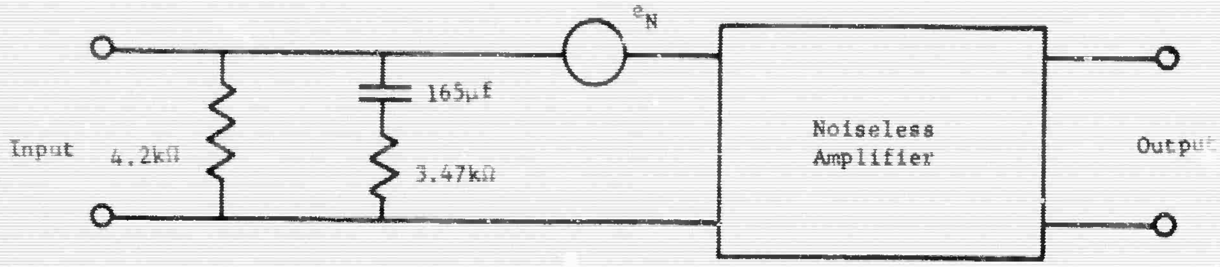

Fig. 18. Noise Equivalent Circuit of Electrotech Model SPA-1-1 Amplifier.

In this equivalent circuit the spectrum level of the noise voltage generator e_N is that presented in Fig. 15 and the gain of the noiseless amplifier is that shown in Fig. 16.

D. REVIEW OF NOISE CHARACTERISTICS OF TEXAS INSTRUMENTS MODEL RA-2 AMPLIFIER AND GEOTECH MODEL 4300 GPTA

Measured noise performance of the Model RA-2 Amplifier and the Model 4300 GPTA have been presented in Semiannual Technical Summary No. 5 and Technical Summary Report for June 1961 to September 1962 respectively. The results of the noise measurements on these amplifiers have been recalculated to conform with the change in noise performance presentation adopted in this report.

1. Noise Performance – Geotech Model 4300 GPTA

The noise characteristics of the Model 4300 GPTA may be represented by the equivalent circuit of Fig. 19. The input circuit parameters are shown in Fig. 19. The amplifier excess noise

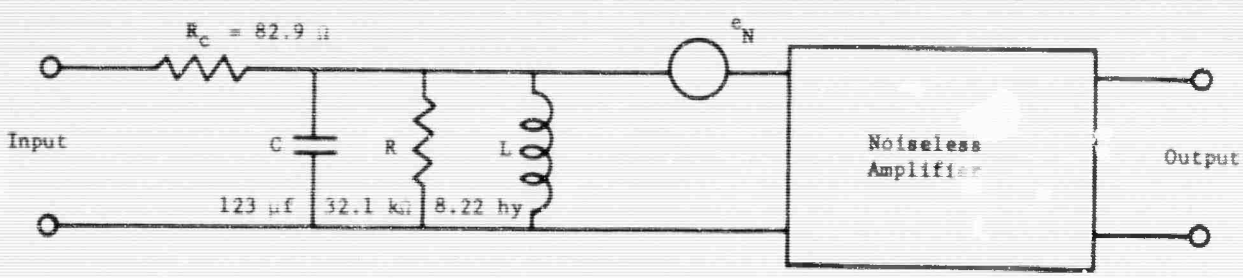

Fig. 19. Noise Equivalent Circuit of Geotech Model 4300 Galvanometer-Phototube Amplifier.

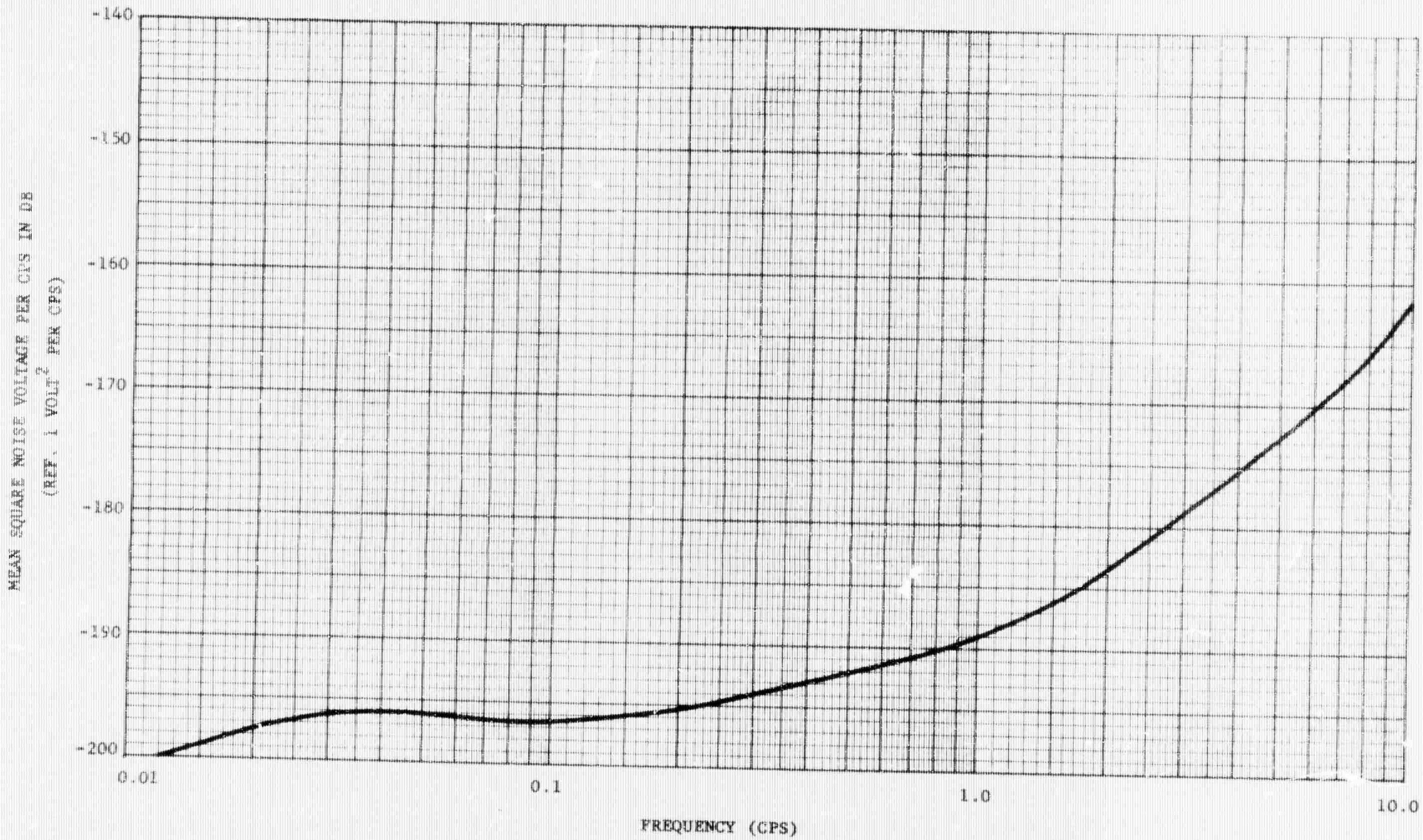

Fig. 20. Equivalent Excess Noise Voltage Spectrum of Geotech Model 4300 Galvanometer-Phototube Amplifier.

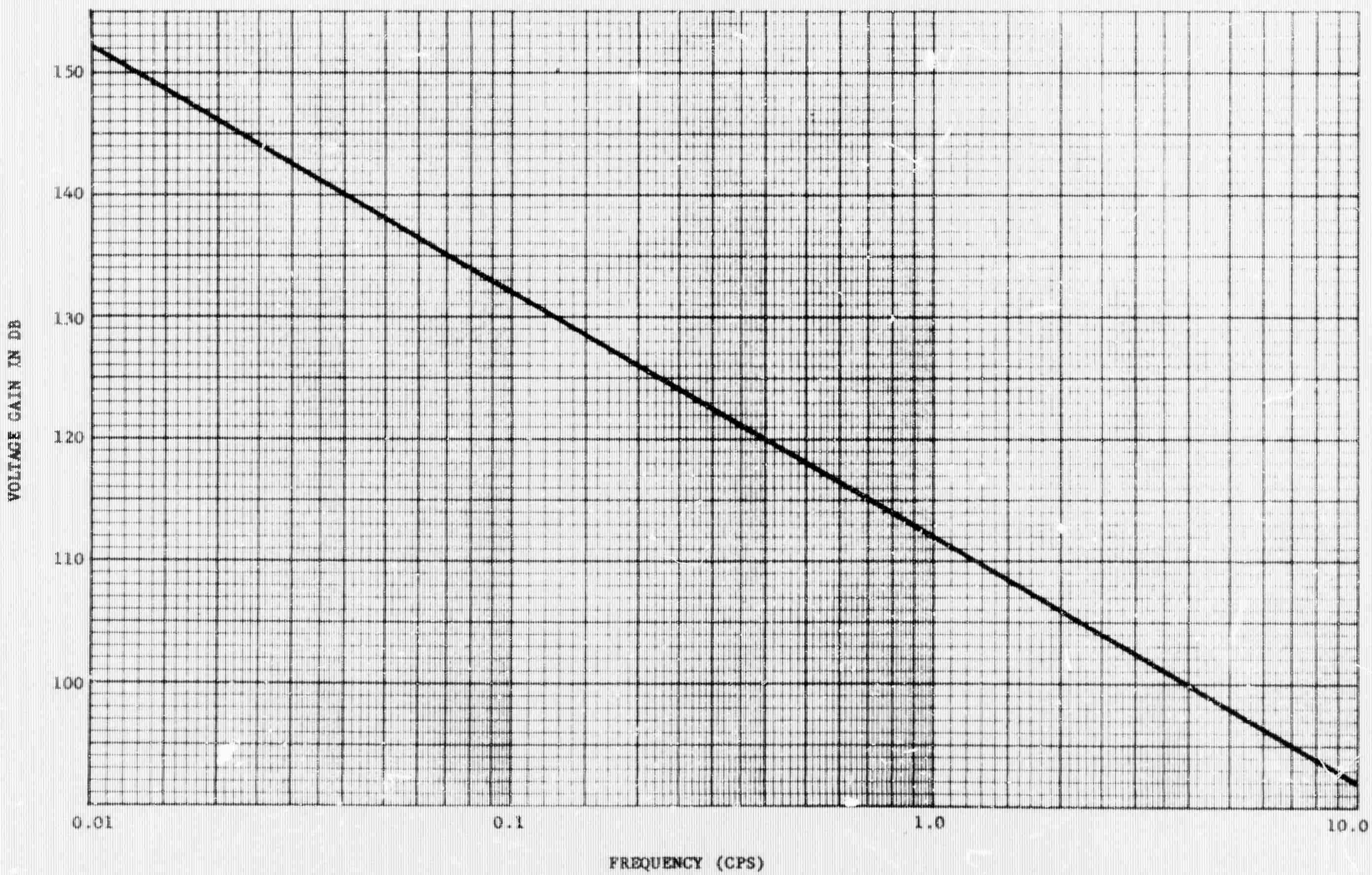

Fig. 21. Noiseless Amplifier Gain for Geotech Model 4300 Galvanometer-Phototube Amplifier.

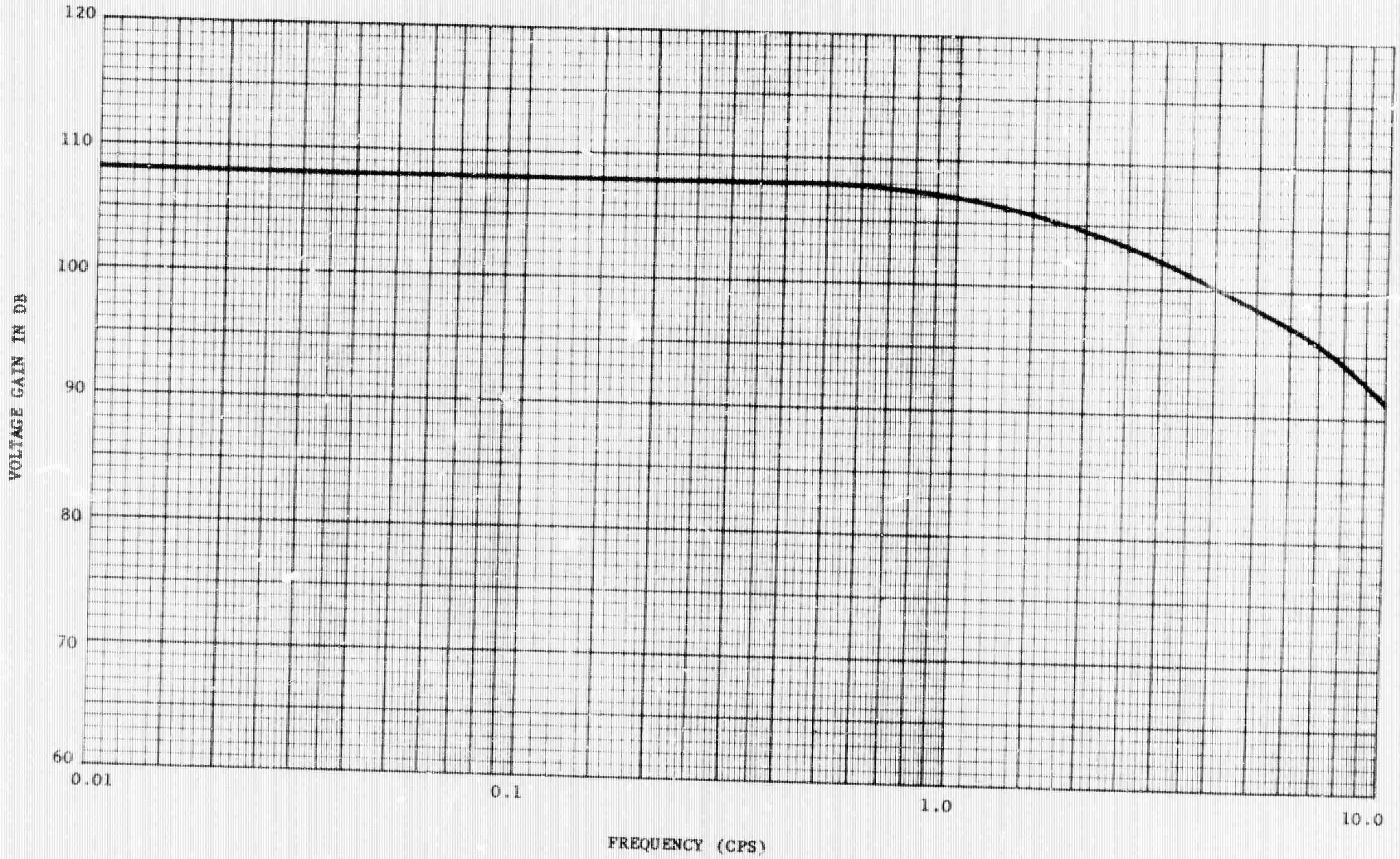

Fig. 22. Geotech Model 4300 Galvanometer-Phototube Amplifier Gain.

spectrum of the equivalent noise voltage source e_N for this amplifier is given in Fig. 20. The gain of the noiseless amplifier is presented in Fig. 21. The overall gain of the Geotech Model 4300 GPTA is shown in Fig. 22.

2. Noise Performance – Texas Instruments Model RA-2 Amplifier

The equivalent circuit of the Model RA-2 Amplifier for noise performance representation is shown in Fig. 23. The input impedance of this amplifier is made up of the shunt combination

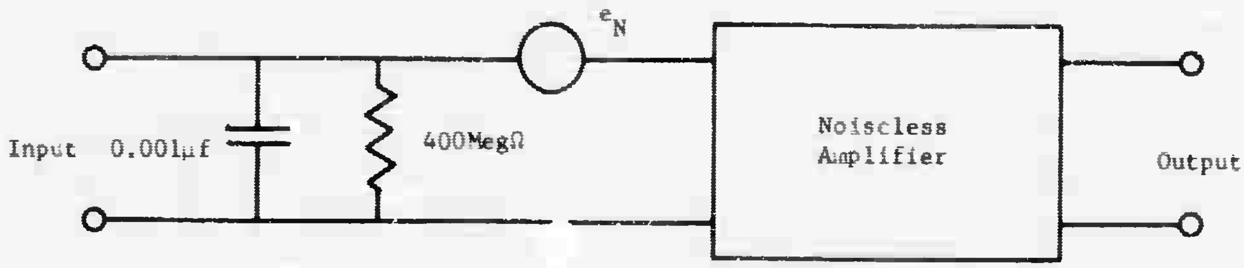

Fig. 23. Noise Equivalent Circuit of Texas Instruments Model RA-2 Amplifier.

of the resistance and capacitance shown in Fig. 23. The noise spectrum of the equivalent input excess noise voltage source e_N of the amplifier and the gain of the noiseless amplifier are presented in Fig. 24 and Fig. 25 respectively.

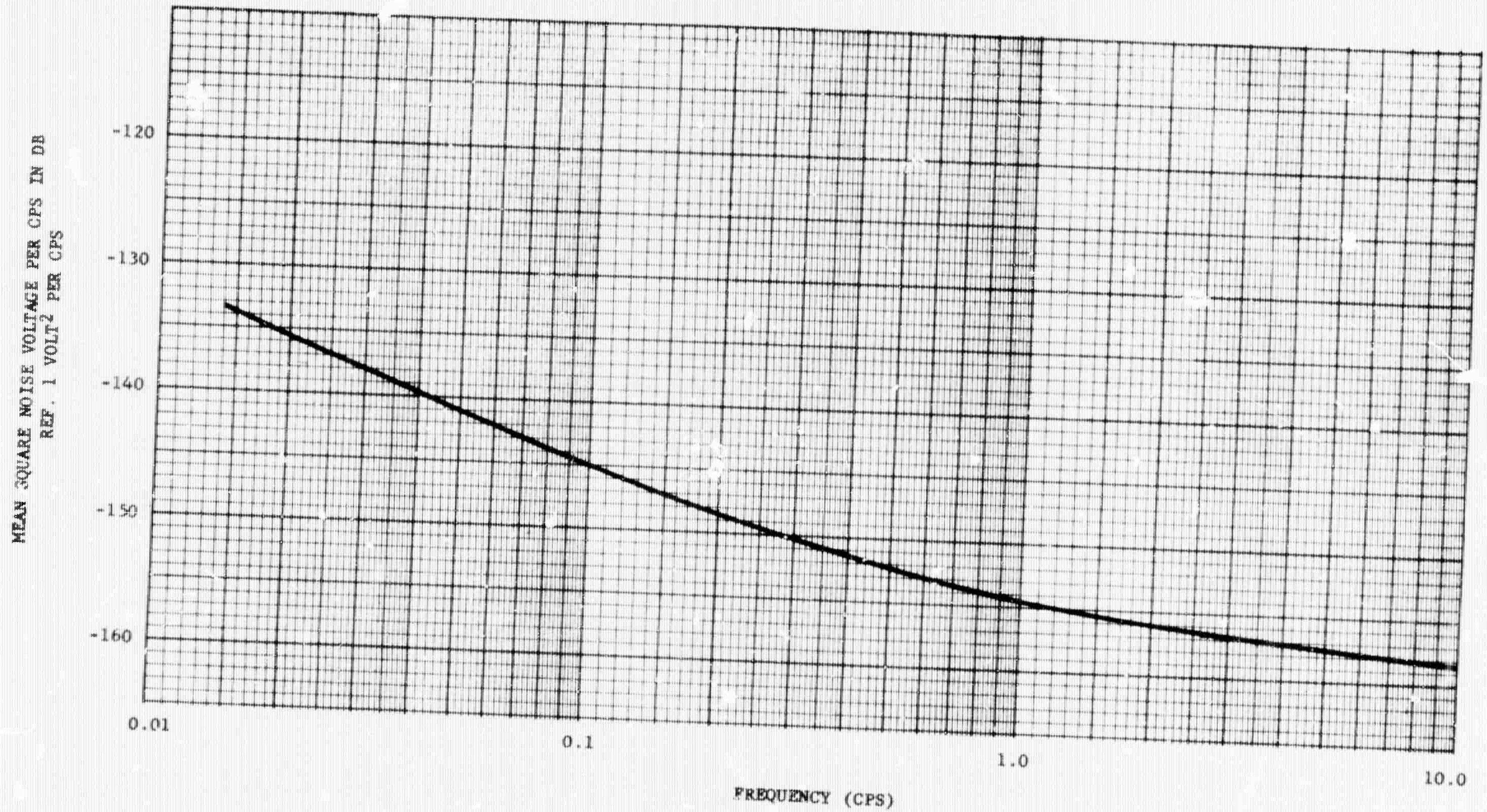

Fig. 24. Equivalent Input Excess Noise Voltage Spectrum of Texas Instruments Model RA-2 Amplifier.

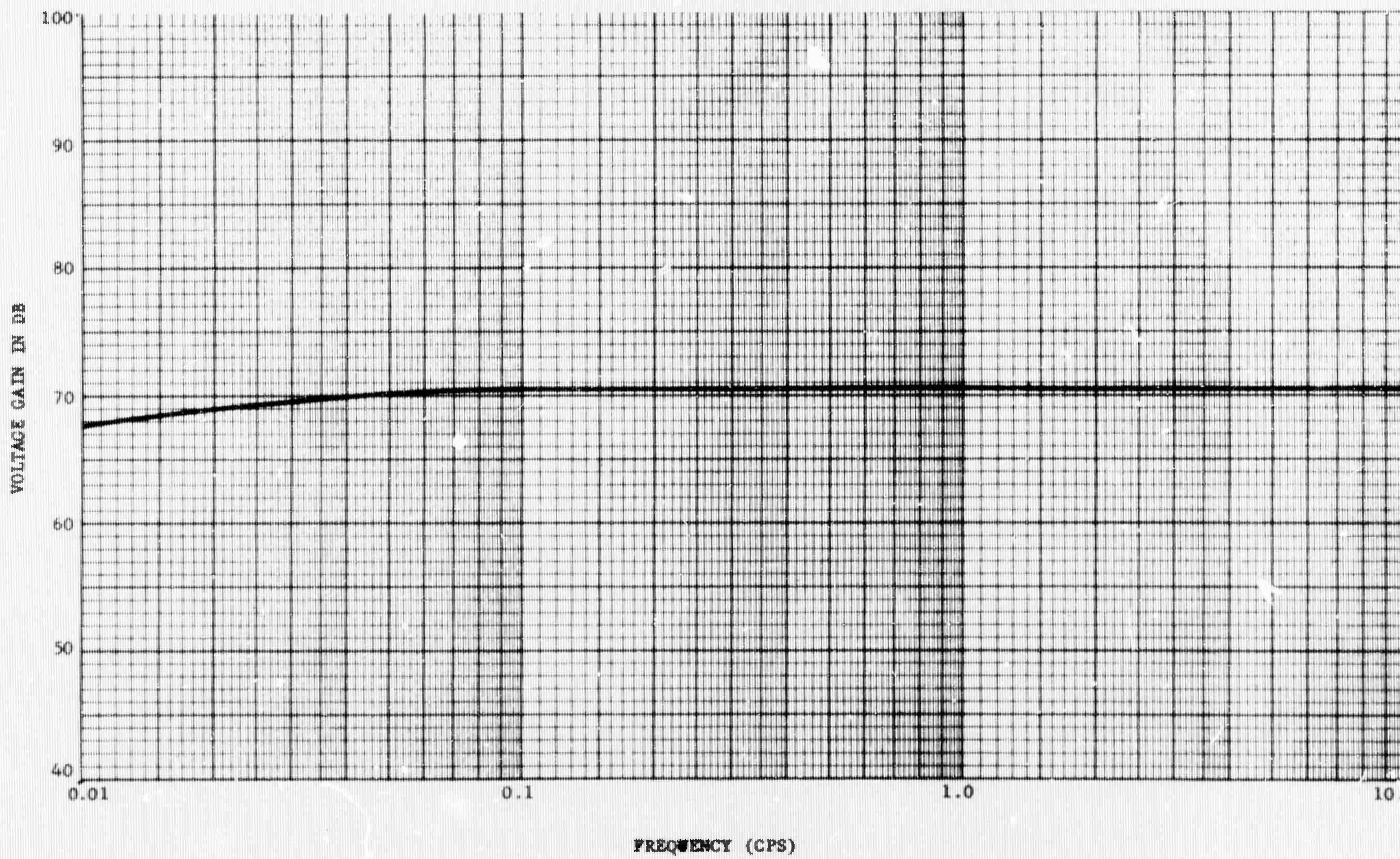

Fig. 25. Texas Instruments Model RA-2 Amplifier Gain.

IV. COUPLING SEISMOMETERS AND AMPLIFIERS FOR MAXIMUM SIGNAL TO NOISE RATIO

After the designer of a seismic detector system has chosen the inertial mass, resonant frequency, and damping factor of his seismometer and has picked a low-noise amplifier with the proper size, battery drain, cost, etc. to meet the requirements of his application, he is faced with the problem of matching the impedance of the two devices in order to achieve the best signal to noise ratio. The amplifier will probably have a fixed input impedance, at least at its maximum sensitivity. Seismometer manufacturers, however, usually offer a range of transducer coil resistances and generator constants and will make special coils on order. It is the purpose of this section to show how the choice of transducer coil resistance affects the signal to noise ratio and to lay down some guidelines for choosing the transducer parameters.

C. J. Byrne[1] pointed out in his article on instrument noise in seismometers that maximum signal to noise ratio is obtained when the coil resistance and generator constant of the seismometer are chosen so that the input resistance of the amplifier provides the only external damping. This result is correct but it is difficult to realize in practice for amplifiers with extremely high input resistances. The Texas Instruments RA-2 Amplifier for example has an input resistance in excess of 400 megohms. Moving coil transducers have been fabricated with coil resistances of roughly 0.5 megohms but are extremely difficult to make because of the miles of very fine wire required. A moving-coil transducer which will permit the seismometer to be damped properly by 400 megohms is certainly not a practical one. Therefore, it becomes important to investigate the manner in which signal to noise ratio varies as the transducer coil resistance and generator constant are varied.

Following the analytic procedures established in earlier reports the mechanical elements of the seismometer may be expressed in terms of the equivalent electrical components which appear to be present when viewed from the electrical terminals of the transducer. The electrical equivalent circuit which will be considered is shown in Fig. 26.

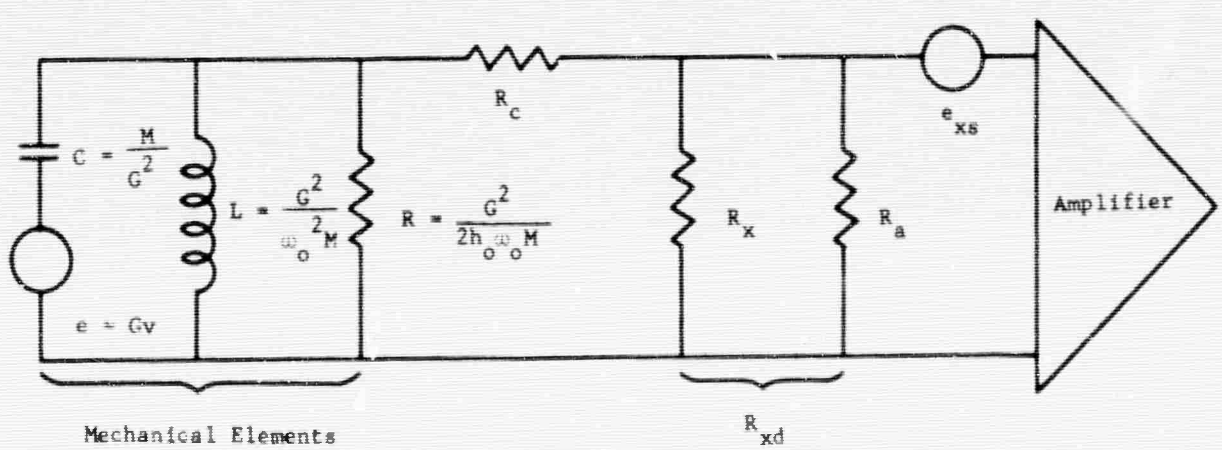

Fig. 26. Electrical Equivalent Circuit of Seismometer and Amplifier.

In this figure the parameters are:

- v = velocity of ground motion relative to an inertial reference
- G = generator constant of the transducer
- M = inertial mass of the seismometer
- f_o = undamped resonant frequency of the seismometer ($\omega_o = 2\pi f_o$)
- C = capacitance, equivalent to inertial mass
- L = inductance, equivalent to spring compliance
- R = equivalent damping resistance present when the transducer is open-circuited
- h_o = open-circuited damping factor of the seismometer
- R_c = transducer coil resistance
- R_x = an external damping resistance placed in shunt with the amplifier input terminals
- R_a = input resistance of the amplifier
- R_{xd} = total external damping resistance (consisting of R_x and R_a in parallel)
- e_{xs} = rms amplifier excess noise referred to the input terminals of the amplifier.

In order to simplify the analysis the inductance of the transducer coil has been neglected, the amplifier input impedance is assumed to be purely resistive within the passband, and the internally generated amplifier noise has been represented as a single noise voltage source at the input. These constraints have been discussed in earlier reports and are reasonable approximations for most seismometers and amplifiers. The input impedance of galvanometer type amplifiers, however, is not resistive at frequencies near the galvanometer resonance.

The approach which will be used is to determine the manner in which the signal and thermal noise voltages at a particular frequency vary as a function of coil resistance, R_c, at the input terminals of the amplifier. The signal will then be compared with the total noise voltage, consisting of the sum of the mean square values of thermal noise and amplifier excess noise.

A familiar difficulty presents itself at this point. While the signal is at a discrete frequency the noise spectrum is continuous and it is therefore necessary to assign a bandwidth to the noise in order to determine a mean square value. For simplicity it will be assumed that the signal is being compared with the noise in a one cycle band centered about the signal frequency.

In order to present the results as clearly as possible, the derivation of the mathematical relationships has been relegated to an appendix which appears at the end of the report. In the Appendix the following relations are shown to be true for the circuit of Fig. 26:

1. The coil resistance, R_c, is directly proportional to the square of the generator constant, G, if the air gap flux density and volume of the transducer coil are maintained constant as R_c is varied. See section A of the Appendix.

$$G^2 \sim R_c \tag{3}$$

2. The external damping resistance, R_{xd}, consisting of the parallel combination of R_x and the amplifier input resistance, R_a, is directly proportional to R_c until R_{xd} becomes equal to R_a. For higher values of coil resistance it is necessary to insert additional resistance in series with R_c to maintain the damping factor constant. See section B of the Appendix.

$$\begin{aligned} R_{xd} &\sim R_c \text{ for } \left[0 \leq R_{xd} \leq R_a\right] \\ R_{xd} &= R_a \text{ for higher values of } R_c \end{aligned} \tag{4}$$

3. The mean-square signal voltage at the input terminals of the amplifier, $\overline{e_{si}^2}$, is directly proportional to R_c for constant damping factor until the value of R_c for which $R_{xd} = R_a$ is reached. For higher values of R_c the signal voltage becomes inversely proportional to R_c. See section B of the Appendix.

$$\begin{aligned} \overline{e_{si}^2} &\sim R_c \text{ for } \left[0 \leq R_{xd} \leq R_a\right] \\ \overline{e_{si}^2} &\sim \frac{1}{R_c} \text{ for higher values of } R_c \end{aligned} \tag{5}$$

4. The mean-square thermal noise voltage at the input terminals, $\overline{e_{th}^2}$, is directly proportional to R_c until the value for which $R_{xd} = R_a$ is reached. For higher coil resistances the thermal noise remains approximately constant. See section C of the Appendix.

$$\begin{aligned} \overline{e_{th}^2} &\sim R_c \text{ for } \left[0 \leq R_{xd} \leq R_a\right] \\ \overline{e_{th}^2} &= \text{approximately constant for higher values of } R_c \end{aligned} \tag{6}$$

These results are plotted on a log-log scale in Fig. 27.

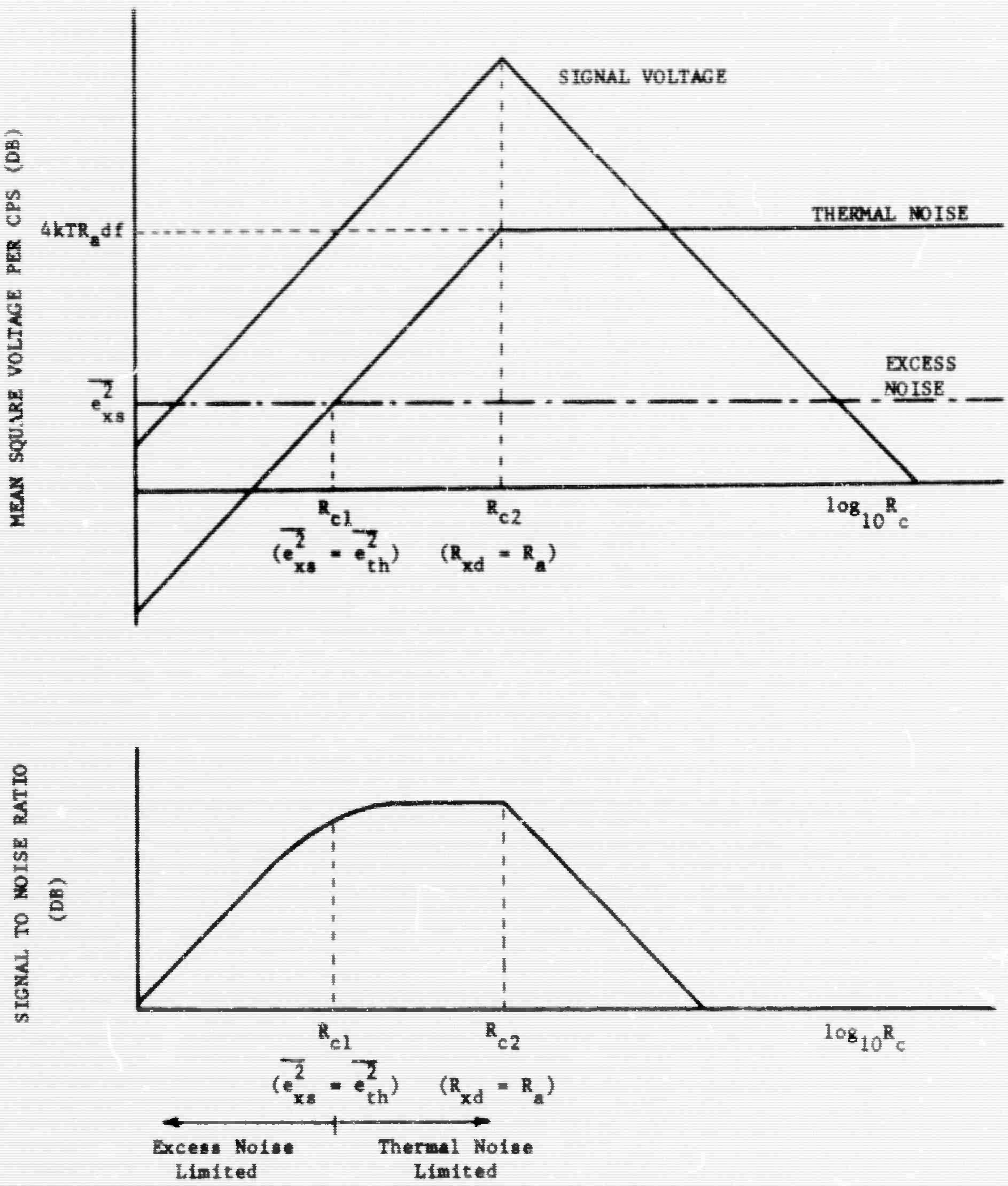

Fig. 27. Signal Voltage, Thermal Noise, Excess Noise and Signal to Noise Ratio vs. R_c.

If the mean square excess noise over the desired passband of the system is less than the mean square thermal noise of the input resistance of the amplifier (as shown in Fig. 27), the signal to noise ratio may be divided into three regions:

1. For low values of R_c the value of R_x must be low in order to damp the seismometer properly. As a consequence, the thermal noise is low and the signal to noise ratio is limited by the amplifier excess noise. Because of the direct relation between R_c and G^2, however, the signal is low and the signal to noise ratio is low.

2. For medium values of R_c, for which the thermal noise exceeds the amplifier excess noise, the signal and thermal noise are both proportional to R_c and the signal to noise ratio becomes constant.

3. For high values of R_c, for which the external damping resistance required to damp the seismometer becomes higher than the amplifier input resistance, it becomes necessary to place external resistance in series with R_c and the signal drops off as R_c increases. The thermal noise tends to flatten off, however, and the signal to noise ratio decreases. (At the resonant frequency of the seismometer the thermal noise becomes absolutely constant over this range).

Therefore, the maximum signal to noise ratio is achieved at the value of R_c for which the required external damping resistance is the input resistance of the amplifier but if the mean square excess noise is less than the thermal noise of the input resistance of the amplifier over the desired passband, it is possible to use a transducer with a lower value of coil resistance without significant loss of signal to noise ratio.

This may be seen by inspection of Fig. 28. In this figure, plots of signal to noise ratio vs. R_c are shown for the following seismometer-amplifier combinations:

1. A Johnson-Matheson Vertical Seismometer driving a Texas Instruments RA-2 Parametric Amplifier.

2. A Hall-Sears HS-10-1 Seismometer driving a Texas Instruments RA-2 Parametric Amplifier.

3. A Johnson-Matheson Vertical Seismometer driving an Electrotech SPA-1-1 Amplifier.

4. A Hall-Sears HS-10-1 Seismometer driving an Electrotech SPA-1-1 Amplifier.

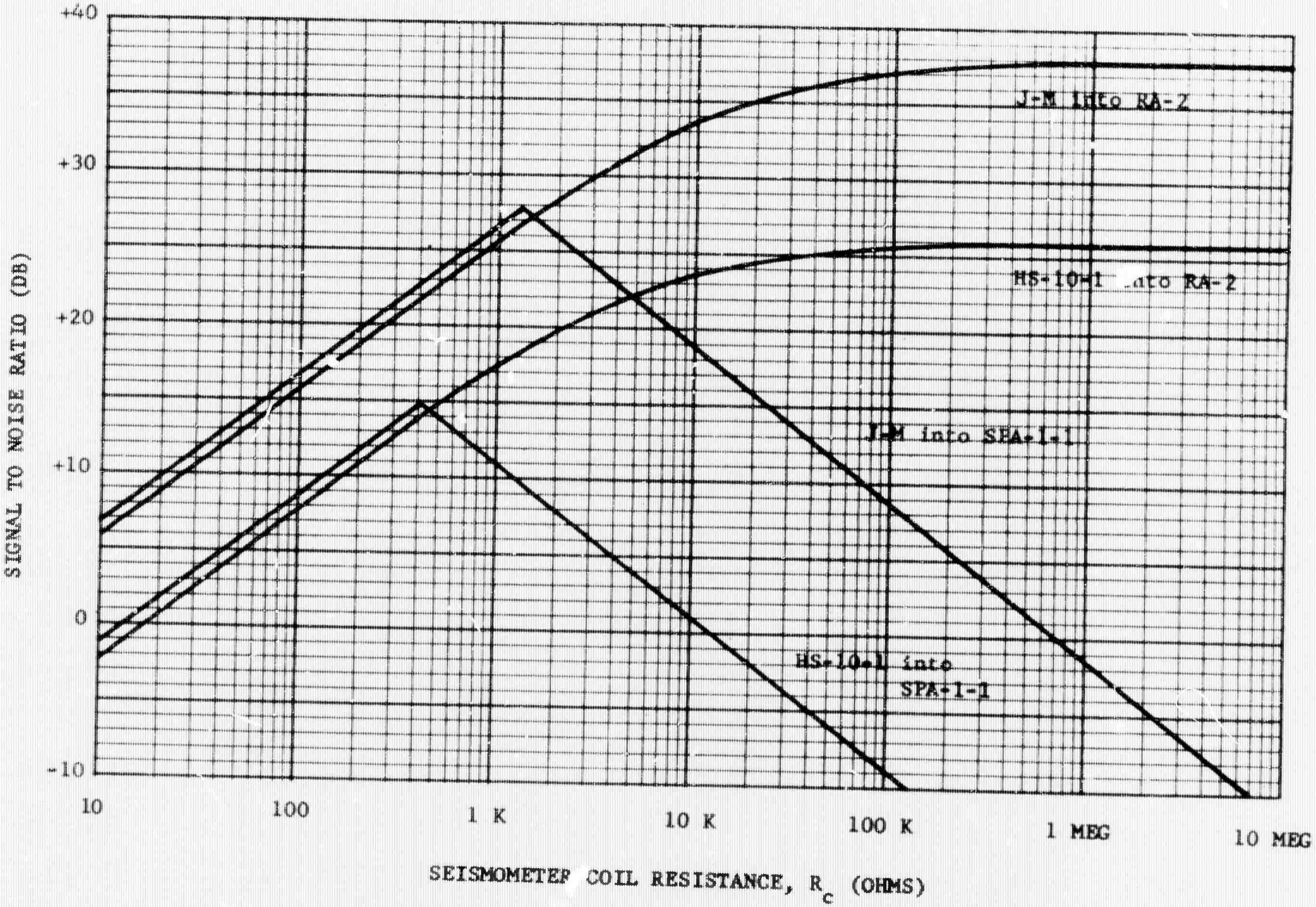

Fig. 28. Signal to Noise Ratio vs. R_c for Four Seismometer-Amplifier Combinations.

The ground velocity assumed for these calculations was 1.0 millimicron/sec at 1.5 cps. This frequency was chosen because it is at about the center of the energy spectrum of P waves. The damping factor is taken to be 0.7 of critical.

Even though the excess noise spectrum of these two amplifiers is approximately equal when referred to the input terminals, they differ greatly in their input resistance. The RA-2 has an input resistance in excess of 400 megohms, while the input resistance of the SPA-1-1 is 1900 ohms.

It may be seen that there is no significant increase in the signal to noise ratio curve for either seismometer driving the RA-2 amplifier for coil resistances above 100K ohms and the signal to noise ratio is down only about 3 db for $R_c = 10K$.

The SPA-1-1 amplifier represents a different situation, however. For this amplifier the excess noise in a one cycle band centered at 1.5 cps is already greater than the thermal noise of the amplifier input resistance. Therefore the signal to noise ratio curve never reaches the plateau shown in Fig. 27 where it becomes limited by the thermal noise. For this amplifier the choice of coil resistance of the seismometer is fairly critical and should be chosen as that value for which the input resistance of the amplifier by itself provides the desired damping.

This resistance is given by the relation

$$R_c = \frac{R_a}{\frac{1}{4\pi f_o (h - h_o) M}\left(\frac{G_o^2}{R_{co}}\right) - 1} \tag{7}$$

where G_o and R_{co} are nominal values of generator constant and coil resistance, respectively, for the seismometer transducer being considered. The ratio G_o^2/R_{co} should be a constant for a particular transducer so long as the air gap flux density and coil volume are fixed.

When working into an SPA-1-1 the Johnson-Matheson seismometer should have a coil resistance of approximately 1300 ohms while the HS-10-1 should have a coil of 430 ohms.

CONCLUSIONS

1. If the amplifier excess noise is greater than the thermal noise of the input resistance of the amplifier over the desired passband, the signal-to-noise ratio of the system is excess noise limited. Further, there is a sharp maximum in the signal to noise ratio when the coil resistance is adjusted so that the seismometer has the desired damping when the external damping is provided solely by the input resistance of the amplifier. This coil resistance may be calculated from Eq. 7.

2. If the amplifier noise is less than the thermal noise of the input resistance of the amplifier over the desired passband, there is a range of transducer coil resistance for which the signal to noise ratio is thermal noise limited and the coil resistance may be chosen at the value for which the excess noise is equal to the thermal noise without serious loss of signal to noise ratio.

3. It would appear, that for amplifiers having resistive input impedances the ratio of amplifier excess noise to the thermal noise of the amplifier input resistance over the desired passband is an important parameter for comparing amplifiers for use with seismic detectors.

V. SIGNAL TO NOISE RATIOS FOR SEVERAL SEISMOMETER-AMPLIFIER COMBINATIONS

The final basis for determining the relative noise performance of a seismometer and an amplifier is in terms of the signal to noise ratio (SNR) appearing at the output of the combination. Calculations of SNR were made for two seismometers and four amplifiers using the amplifier noise characteristics measured. The two seismometers considered were the Geotech Johnson-Matheson Vertical Seismometer and the Hall Sears HS-10-1 Seismometer. SNR calculations were made for these seismometers and for the Electrotech Model SPA-1-1 Amplifier, the Texas Instruments Model RA-2 Amplifier, the United Electrodynamics Model GA-220 Galvanometer-Photosensitive Amplifier, and the Geotech Model 4300 Galvanometer Phototube Amplifier.

The SNR calculations were made for an input ground velocity of 1.0 millimicron/sec at a frequency of 1.5 cps. The bandwidth considered is 1 cps wide about the center frequency of 1.5 cps. The seismometer damping used was 0.7 times critical. The seismometer windings were considered to be variable under the same constraints used in the previous section of this report on coupling seismometers and amplifiers for maximum signal to noise ratio. The SNR was calculated for the optimum conditions as discussed in the previous section of the report for the systems using the SPA-1-1 Amplifier and the RA-2 Amplifier. Optimum conditions are more difficult to specify for galvanometer type amplifiers. As a consequence, calculations of SNR using the GA-220 Amplifier and the Model 4300 GPTA were made using the conditions given above and in which the total external seismometer damping was supplied by the amplifier input impedance. For these two amplifiers the seismometer damping of 0.7 of critical was determined for the blocked galvanometer condition, i.e., the external damping resistance equal to the galvanometer winding resistance. The results of the calculations should be representative of the SNR performance of these last two amplifiers.

The results of the SNR calculations are shown in the bar graph of Fig. 29. Summarizing the results, the RA-2 Amplifier, the GA-220 and the Model 4300 GPTA used with the J-M seismometer and used with the HS-10-1 seismometer have similar SNR with a lower SNR for the HS-10-1 than for the J-M. When using the J-M seismometer the Model 4300 GPTA has a slight advantage in SNR while for the HS-10-1 seismometer the RA-2 has a slight advantage. Using either seismometer the SPA-1-1 has a somewhat poorer SNR than the other three amplifiers.

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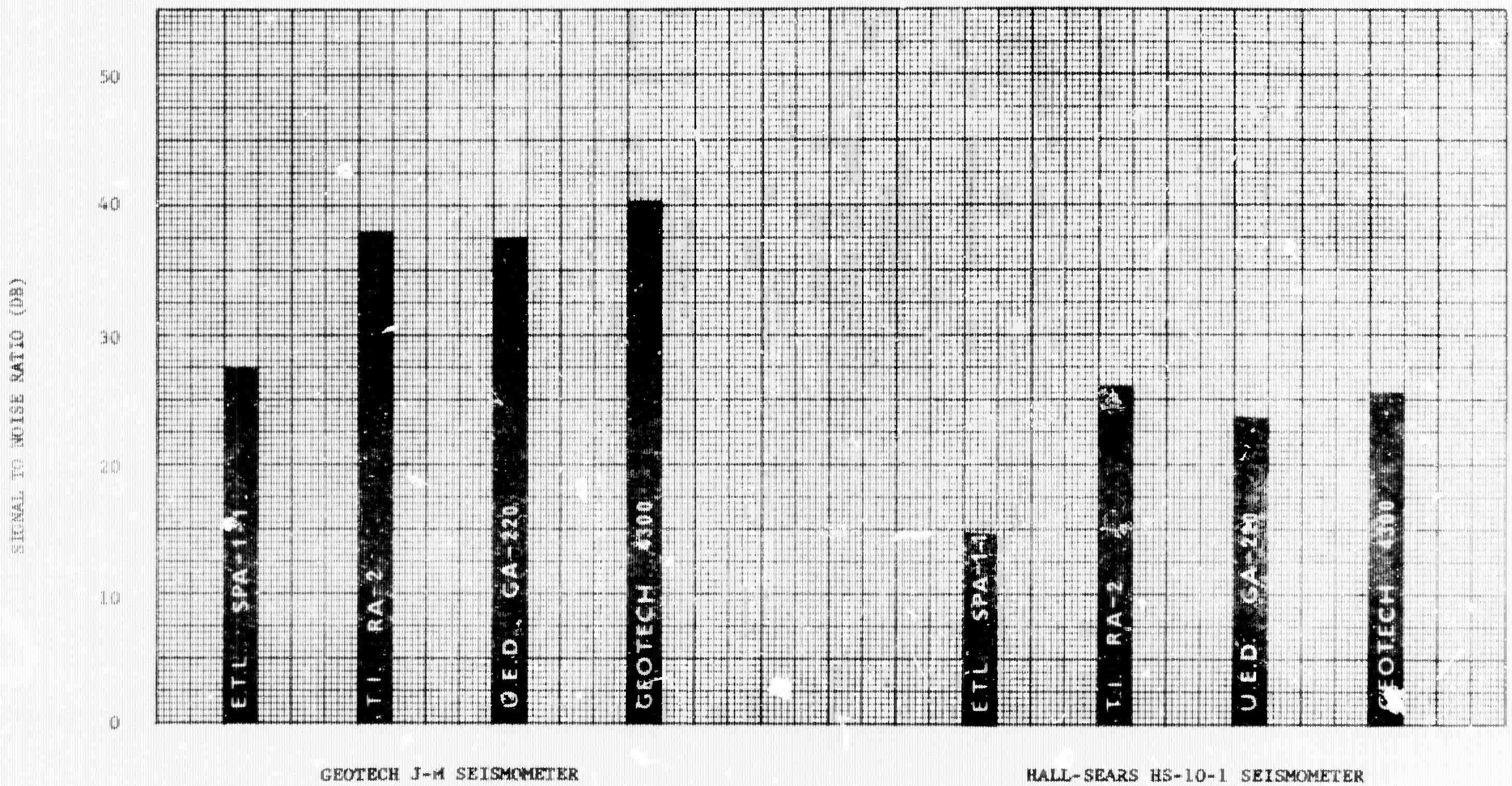

Fig. 29. Signal to Noise Ratio of Seismometer-Amplifier Combinations.

VI. VISUAL DETECTION OF SIGNAL IN THE PRESENCE OF NOISE

INTRODUCTION

In order to determine the limiting sensitivity of a seismic detection system as imposed by its noise performance some criteria relating the minimum detectable signal to the noise must be used. A study of the available information on this subject indicated that very little was known about how the eye-brain complex functions to detect signals in noise or what the important parameters affecting the visual detection process are. Because of the insufficient information available on this subject and because of its importance to the primary objective of this contract a study was undertaken to develop criteria for the detectability of a signal in the presence of noise and to improve our understanding of the functioning of the visual detection process in general.

A. WOLF'S CRITERION

One of the very few references found in the literature on the problem of visual detection of a signal in noise was in a paper published by Wolf[4] in 1942. In this paper Wolf states that his experience with galvanometers indicated that an rms signal to rms noise ratio (SNR) equal to four or five times is required to visually detect the signal with certainty. The actual number used by Wolf in his paper was SNR = $\sqrt{20}$. Since Wolf's criterion has received a fair amount of acceptance and use in the seismic field, special consideration was given to it.

A series of tests conducted specifically using Wolf's criterion indicate that Wolf's criterion for detectability is extremely conservative and that the visual detection process is more sophisticated than originally indicated by Wolf. Examples of these tests and an outline of the tests used were presented in Semiannual Technical Report No. 5.

B. VISUAL FILTERING IN THE DETECTION PROCESS

Results of tests conducted and presented in Semiannual Technical Report No. 5 and other tests conducted subsequently indicate that the visual detection process is not very much different from the aural detection process, in that both detection processes can effectively discriminate against unwanted signals or noise outside of some band of interest. Essentially this means that the eye-brain complex has the ability to filter the total signal and effectively not see or discriminate against noise with a frequency spectrum outside the pass-band of this visual filter. Since, in the visual process as it relates to the reading of a permanent chart recording, the concept of time does not really exist it will be necessary to describe the characteristics of the visual filtering ability of the eye in terms of what is actually presented to the

[4] Alfred Wolf – The Limiting Sensitivity of Seismic Detectors. J. of General and Applied Geophysics, Vol. VIII, #2, April 1942.

viewer, i.e., displacement rather than time. Therefore the visual filter characteristics will have to be given in terms of cycles/mm rather than cycles/sec.

Quantitative analysis of the tests indicate that the ability of the visual detection process to discriminate against unwanted signals or noise can be represented by a filter with a geometric mean center frequency of 0.2 cycles/mm and an effective bandwidth of 3 octaves. Furthermore, for a sinusoidal signal to be detectable withi the bandpass of the filter a signal to noise ratio (SNR) of 2 is required. This means, for example, that if a chart recording of a 1.0 cycle/sec signal is made with a chart speed of 5mm/sec, the effective filtering ability of the visual detection system can be represented by a filter with a geometric mean center frequency of 1.0 cps and an effective bandwidth of approximately 2.5 cps.

If, for the above example, the noise is centered about 1.0 cps and has a relatively uniform spectral distribution, a SNR of 2 will be required for detectability provided the bandwidth of the noise is less than 2.5 cps. However, if the bandwidth of the noise is greater than 2.5 cps the required SNR for detectability will be less than 2 because of the ability of the eye to discriminate against the noise outside of the 2.5 cps range.

In general, these results indicate that for a relatively uniform noise spectrum level and a signal to noise ratio of 2, (SNR = 2), a sinusoid will be detectable independent of the actual bandwidth of the noise recorded. Therefore, for a rule of thumb type crit ion for detectability, a signal to noise ratio of 2, (SNR = 2), could be used.

C. EFFECT OF SIGNAL DURATION ON DETECTABILITY

A series of tests was conducted using sinusoidal bursts of various durations in order to examine the effect of signal duration on detectability. A sample trace similar to the traces used in the actual test is shown in Fig. 30. The operating conditions for this sample trace were chosen to be identical to the conditions assumed for the example given in the previous section of this report, i.e., a chart speed of 5 mm/sec and a test signal of 1.0 cps. A Random Noise Generator, bandlimited so as to have an essentially flat spectrum level of 10 mv/$\sqrt{\text{cycle}}$ between 0.1 cps and 16 cps, and a 31.5 mv sinusoid were used in making the sample trace. These numbers were chosen so that the SNR within the 2.5 cps bandwidth of the visual filter would be equal to 2, (SNR = 2), with sufficient noise signal existing outside the bandwidth of the visual filter to demonstrate the effect of the filter. The actual SNR for the trace presented in Fig. 30 is 1, (SNR = 1). As can be seen from the sample trace: the signal 5 cycles in duration can be detected with a fair degree of certainty; 3 cycles is more difficult to detect; 2 cycles, in general, is difficult to detect; and signals 1 cycle in duration are very difficult to detect with any degree of certainty.

It should be noted that for the tests with sinusoidal bursts the rms value of the test signals is *defined* as b ng equal to 0.707 times the peak value of the signal, i.e., the rms

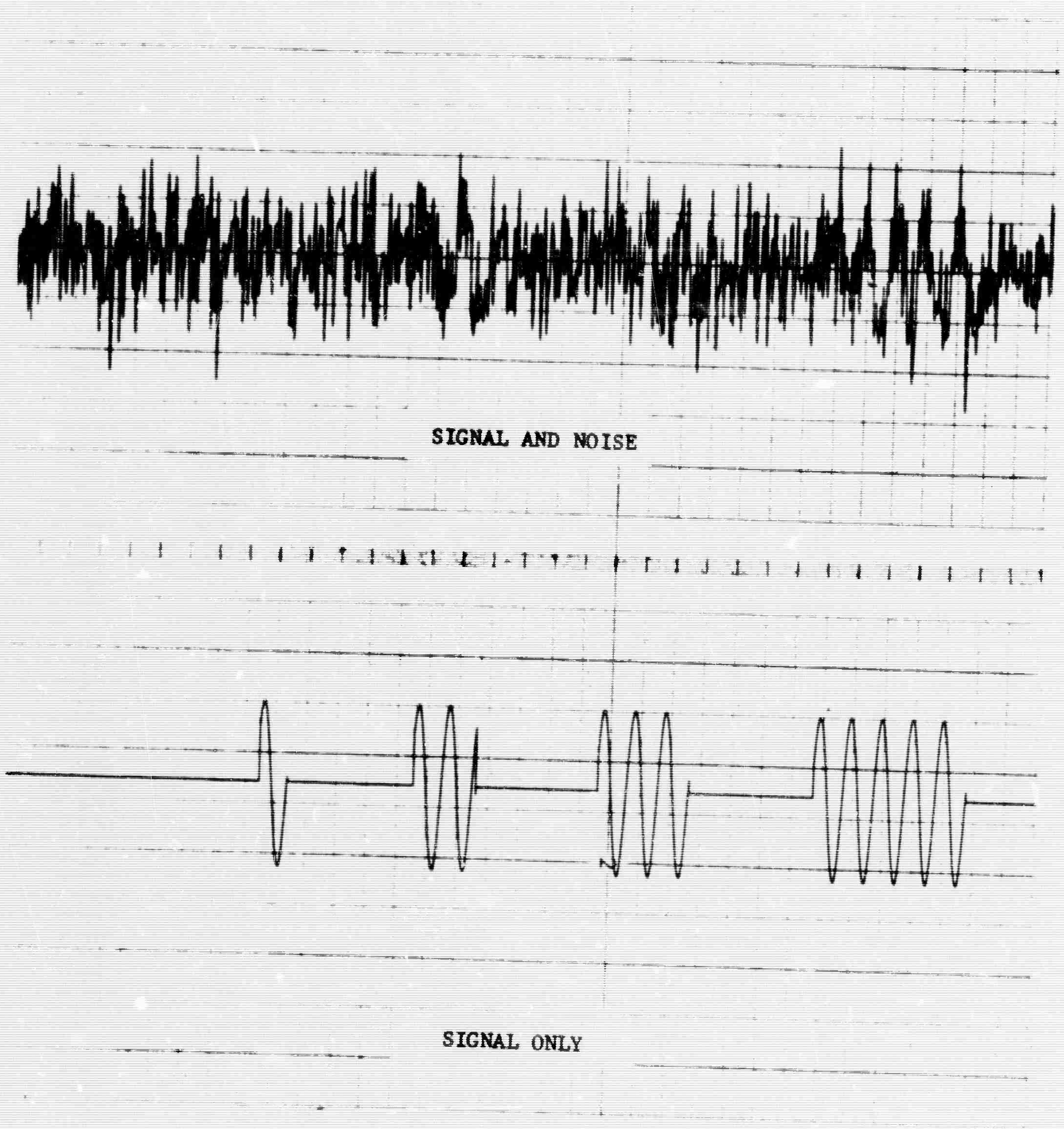

Fig. 30. Sample Trace of Sinusoidal Bursts in 0.1 - 10.0 CPS
Band Limited Noise. SNR = 1.0.

value of the sinusoidal bursts is *defined* as being equal to the rms value of a continuous sinusoid of the same amplitude. Defining the rms value of a sinusoidal burst is necessary because the actual rms value of any non-continuous signal is mathematically equal to zero, and this particular manner of defining the signal is useful because it allows the detectability criteria established for continuous sinusoids to be applied to sinusoidal bursts.

The results of the actual tests conducted indicate that the criteria established for the filtering ability of the visual detection system and the required SNR for detectability are applicable to sinusoidal signals 2 to 3 cycles or greater in duration.

APPENDIX TO SECTION IV

A. THE RELATION BETWEEN COIL RESISTANCE AND GENERATOR CONSTANT

The resistance, R_c, of the transducer coil is given by

$$R_c = \rho \frac{an}{A} = \rho \frac{(an)^2}{V_c} \qquad \text{(A-1)}$$

where ρ = resistivity of the transducer coil conductor

a = mean length of turn

n = number of turns

A = cross-sectional area of the conductor

V_c = total volume of the conductor

The generator constant, G, is given by

$$G = Ban \qquad \text{(A-2)}$$

where B is the air gap flux density. Combining equations (A-1) and (A-2)

$$R_c = \frac{\rho}{B^2 V_c} G^2 \qquad \text{(A-3)}$$

If the flux density, coil volume and conductor resistivity are kept constant as R_c is varied, then

$$\frac{G^2}{R_c} = \frac{G_o^2}{R_{co}} = \frac{\rho}{B^2 V_c} \qquad \text{(A-4)}$$

where G_o and R_{co} are nominal values of generator constant and coil resistance which are usually available from the seismometer manufacturer.

B. SIGNAL VOLTAGE

The manner in which the signal voltage varies as coil resistance R_c is varied must now be determined. The electrical equivalent circuit of the seismometer and amplifier is reproduced below for reference.

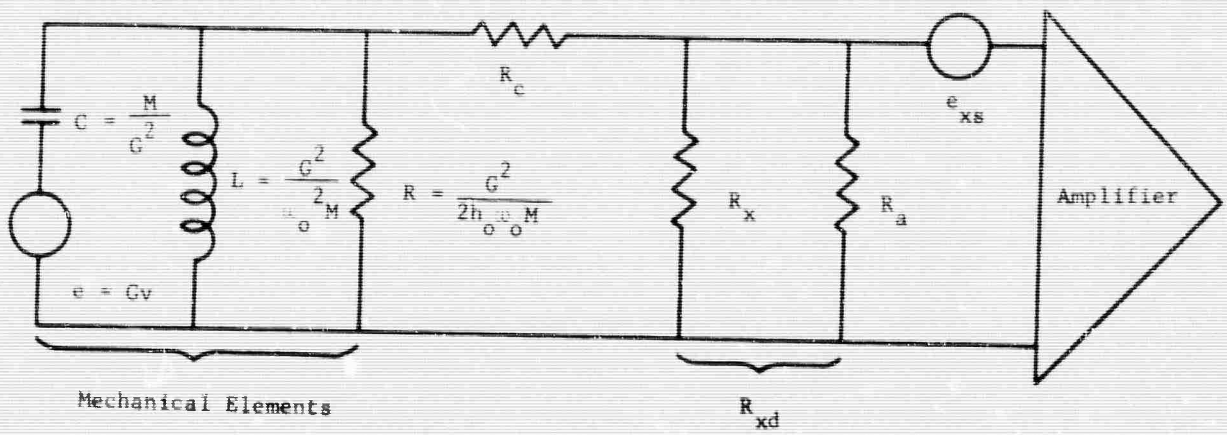

Fig. A-1. Electrical Equivalent Circuit of Seismometer and Amplifier.

By straightforward analysis of this circuit the mean square signal voltage at the amplifier terminals is given by

$$\overline{e_{si}^2} = G^2 v^2 \left(\frac{R_{xd}}{R_c + R_{xd}}\right)^2 \frac{\omega^4}{(\omega_o^2 - \omega^2)^2 + \omega^2 \frac{G^4}{M^2}\left(\frac{1}{R} + \frac{1}{R_c + R_{xd}}\right)^2} \qquad \text{(A-5)}$$

But the damping factor of the seismometer, h, may be defined by

$$h = \frac{G^2}{2\omega_o M}\left(\frac{1}{R} + \frac{1}{R_c + R_{xd}}\right) = h_o + \frac{G^2}{2\omega_o M (R_c + R_{xd})} \qquad \text{(A-6)}$$

Making this substitution into Eq. (A-5)

$$\overline{e_{si}^2} = G^2 v^2 \left(\frac{R_{xd}}{R_c + R_{xd}}\right)^2 \frac{\omega^4}{(\omega_o^2 - \omega^2)^2 + 4h^2 \omega_o^2 \omega^2} \qquad \text{(A-7)}$$

For notational convenience, let

$$F_1(\omega) = \frac{\omega^4 v^2}{(\omega_o^2 - \omega^2)^2 + 4h^2 \omega_o^2 \omega^2} \qquad \text{(A-8)}$$

Then the m an square signal voltage is given by

$$\overline{e_{si}^2} = G^2\left(\frac{R_{xd}}{R_c + R_{xd}}\right)^2 F_1(\omega) \tag{A-9}$$

If the damping factor and seismometer resonant frequency are fixed then $F_1(\omega)$ is a constant for a particular signal frequency.

From Eq. (A-6)

$$R_c + R_{xd} = \frac{G^2}{2\omega_o(h - h_o)M} \tag{A-10}$$

But if the air gap flux density and coil volume of the transducer are constant, G^2 is given by Eq. (A-4) as

$$G^2 = \frac{G_o^2}{R_{co}} R_c \tag{A-11}$$

Substituting this relation into Eq. (A-10)

$$R_{xd} = \left[\frac{G_o^2}{2\omega_o(h - h_o)MR_{co}} - 1\right] R_c \tag{A-12}$$

Combining Equations (A-11) and (A-12) with Eq. (A-9)

$$\overline{e_{si}^2} = \frac{G_o^2}{R_{co}}\left[1 - \frac{2\omega_o(h - h_o)MR_{co}}{G_o^2}\right] F_1(\omega) R_c \tag{A-13}$$

Therefore, the mean square signal voltage at the amplifier input terminals is directly proportional to R_c. Eq. (A-12) shows that the total damping resistance is also proportional to R_c. However, an upper limit on R_{xd} is reached when it becomes equal to R_a, the amplifier input resistance. The coil resistance at which this condition is reached may be obtained from Eq. (A-12) if R_{xd} is placed equal to R_a.

$$R_{c_2} = \frac{R_a}{\left[\frac{G_o^2}{2\omega_o(h - h_o)MR_{co}}\right]} \tag{A-14}$$

If the coil resistance is increased above this value then it becomes necessary to add additional resistance, R_c', in series with the transducer coil in order to keep the damping factor constant. The mean square signal voltage then becomes, from Eq. (A-9)

$$\overline{e_{si}^2} = \frac{G_o^2}{R_{co}} \left(\frac{R_a}{R_c + R_c' + R_a} \right)^2 F_1(\omega)\, R_c \tag{A-15}$$

But, for this condition, Eq. (A-10) becomes

$$R_c + R_c' + R_a = \frac{G_o^2}{2\omega_o (h - h_o) M R_{co}}\, R_c \tag{A-16}$$

and the signal voltage is given by

$$\overline{e_{si}^2} = \frac{R_{co} \left[2\omega_o (h - h_o) M R_a \right]^2}{G_o^2}\, \frac{F_1(\omega)}{R_c} \tag{A-17}$$

Therefore, for coil resistances higher than the value for which the amplifier input resistance alone provides the required external damping, the mean square signal voltage is inversely proportional to R_c.

C. THERMAL NOISE AS A FUNCTION OF R_c

Referring again to the equivalent circuit diagram, Fig. A-1, the thermal noise at the input terminals of the amplifier is related to the real part of the impedance between those terminals by the familiar Nyquist noise relation.

$$\overline{e_{th}^2} = 4kT\, R_e\,[Z]\, df \tag{A-18}$$

where k = Boltzmann's constant

T = Absolute temperature

The solution of the circuit for the real part of this impedance involves a fair amount of algebraic manipulation, the result of which is

$$\overline{e_{th}^2} = 4kT\, df \left(\frac{R_c\, R_{xd}}{R_c + R_{xd}} \right) \left[1 + \frac{R_{xd}}{R_c}\; \frac{\left(1 - \frac{h_o}{h}\right) 4h^2\, \omega_o^2\, \omega^2}{(\omega_o^2 - \omega^2)^2 + 4h^2\, \omega_o^2\, \omega^2} \right] \tag{A-19}$$

Again for notational convenience, let

$$F_2(\omega) = \frac{4h^2 \omega_o^2 \omega^2}{(\omega_o^2 - \omega^2)^2 + 4h^2 \omega_o^2 \omega^2} \qquad \text{(A-20)}$$

At the seismometer resonance frequency $F_2(\omega) = 1.0$. Also $F_2(\omega)$ is very nearly equal to unity at frequencies near the seismometer resonance for normally encountered damping factors in the range 0.7 to 1.0. Making this substitution

$$\overline{e_{th}^2} = 4kT\ df \left(\frac{R_c R_{xd}}{R_c + R_{xd}}\right) \left[1 + \frac{R_{xd}}{R_c} \left(1 - \frac{h_o}{h}\right) F_2(\omega)\right] \qquad \text{(A-21)}$$

Substituting Eq. (A-12), which is valid, as before, for values of R_c smaller than the value for which $R_{xd} = R_a$

$$\overline{e_{th}^2} = 4kT\ df \left[1 - \frac{2\omega_o (h - h_o) M R_{co}}{G_o^2}\right] \left[1 + \left(\frac{G_o^2}{2\omega_o h M R_{co}} - 1 + \frac{h_o}{h}\right) F_2(\omega)\right] R_c \qquad \text{(A-22)}$$

The thermal noise is, therefore, directly proportional to R_c for this case.

For values of R_c larger than R_{c_2} defined by Eq. (A-14)

$$\overline{e_{th}^2} = 4kT\ df \frac{(R_c + R_c') R_a}{R_c + R_c' + R_a} \left[1 + \frac{R_a}{R_c + R_c'} \left(1 - \frac{h_o}{h}\right) F_2(\omega)\right] \qquad \text{(A-23)}$$

Substituting Eq. (A-16) into this expression

$$\overline{e_{th}^2} = 4kT\ df\ R_a \left\{1 - \frac{2\omega_o (h - h_o) M R_{co} R_a}{G_o^2 R_c} \left[1 - \left(1 - \frac{h_o}{h}\right) F_2(\omega)\right]\right\} \qquad \text{(A-24)}$$

At frequencies near the seismometer resonance $F_2(\omega)$ becomes approximately equal to unity and

$$\overline{e_{th}^2} \approx 4kT\ df\ R_a \left[1 - \frac{2\omega_o (h - h_o) M R_{co} R_a}{G_o^2 R_c} \frac{h_o}{h}\right] \qquad \text{(A-25)}$$

If the seismometer open-circuit damping factor h_o is small

$$\overline{e_{th}^2} \approx 4kT\ df\ R_a$$

Therefore the mean square thermal noise remains essentially constant for very large values of R_c.

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